A New Interpretation of Quantum Theory — Time as Hidden Variable

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Using 2 more time variables as the quantum hidden variables, we derive the equation of Dirac field under the principle of classical physics, then we extend our method into the quantum fields with arbitrary spin number. The spin of particle is shown naturally as the topological property of 3-dimensional time + 3-dimensional space . One will find that the quantum physics of single particle can be interpreted as the behavior of the single particle in 3+3 time-space .

Among all of the questions in quantum hidden variable theory, the crucial one is : quantum physics is an elementary theory of physics, so the quantum hidden variables should be very basic concepts of physics, then what are these variables ? The other important questions about quantum concepts are: i) Spin is the basic property of particles and it is derived from quantum physics, so is there a classical analogue of spin ? ii) The satisfactory hidden variable theory should be a single particle theory, is that possible for a single particle theory to recover the statistic properties of quantum physics ? iii) Suppose we can get a single particle theory under the principle of classical physics, how can we interpret the non-local result of Bell's inequality ? iv) Why do we have uncertainty relation? Finally, could the new interpretation give us more knowledge than quantum physics does? There are many tries to answer some of above questions as we can see in the work of Bohm [1] and Holland [2] .

In this paper we will introduce a very interesting observation in quantum hidden variable interpretation method: if we add two more time variables t_θ, t_ϕ as the quantum hidden variables, i.e., 3-dimensional time (t, t_θ, t_ϕ) instead of 1 dimension, we'll find that the motion of single particle under 3+3 time-space posseses the same qualitative behavior as the particle in quantum physics and the spin of particle can be simply interpreted as the topological property of 3-dimensional time. And then we will get the Dirac equation and Bargmann-Wigner equation.

Let us consider the two-slit interference experiment of electrons . We know that in order to get the interference pattern, both slits should have contributions to the result even if we control the electron to be emitted one by one. That means if one tries to trace the trajectory of a single electron, the only conclusion he can get is that the single electron goes through the two splits at the "same time", or in other word: "a particle can show at different places at the same time". Let us express this word in more detail : "a particle can show at two different places at the same time but this particle is still a single particle". It sounds weird, but there is one case that these things can really happen, that is: if and only if there are some hidden time variables which we didn't find. That is to say, the electron passes through the two splits at the same "measure time" but at different "hidden time" .

Consider the case that the time in microcosm is more than 1 dimension — for example 2-dimension, and we use clock to measure time. Since the clock can only measure the 1-dimensional time, i.e., the "length" of time, the information about the "direction" of time (except the forward and backward direction in the same line) is unknown. Let us draw a circle and build the polar coordinate (t,θ) on it, where t is modulus of time and θ is angle . For the two points t_1(t,θ_1), t_2(t,θ_2) on the circle with $r = t$, if we start our measurement from the center t_0, because we can only measure the "length" of time, we will treat t_1,t_2 as the same point . Suppose there is a particle A. At time t_1, the space coordinate of A is (x_1,0,0) and at time t_2 the space coordinate of A is (x_2,0,0), then using the knowledge of 1-dimensional time, we will conclude that a particle A can show at different positions x_1, x_2 at the same time t.

This situation is similar to the situation of two-slit interference of electrons except that the latter has the behavior of a plane wave. Now let us make further explorations: what's the physical meanning of "finding A at position x_0" in classic physics? That means at time t, our apparatus is at position x_0, and at the same time, A is also at position x_0, i.e., our apparatus meets the particle at position x_0 at time t. If the particle only shows at x_0 at time $t_1 \neq t$, and at the mean time, our apparatus only shows at x_0 at time t, we will miss that particle .

Extend this logic to the case of 2-dimensional time. Then the meaning of finding a particle at position x_0 in 2-dimensional time is : at time (t,θ_1) our apparatus is at position x_0, and at the same time (t,θ_1) the particle also is at position x_0, i.e., We meet the particle at x_0 at time (t,θ_1). If the particle only shows at x_0 at time (t,θ_2) where $\theta_2 \neq \theta_1$, and our apparatus only shows at x_0 at time (t,θ_1), then we miss that particle . That is why even though a particle can show at different places at the same time, we can still only find one particle instead of 2 or more (Strictly speaking, here we need the Wave-Package Collapse postulate which will be discussed later). Due to the lack of information of the direction(the angle θ) of time, we can only find the particle by chance even though we have already known that the particle will be at position x_0 at time t . The possibility of finding the particle at position x_0 at time t depends on how many different (t,θ) will show on position x_0. If we let the total portion of that angle θ_i(which passes through x_0 at time t) be divided by 2π, and let the result correspond to the square of amplitude of the wave function of the particle, then we will find that the measurement in the multi-dimensional time of a single particle has the same statistic property as the measurement in quantum physics. Furthermore, if a particle at different θ_i with the same radius t has different energies, then at time t, if we measure energy of the particle, we cannot be sure about which result we will get. The possibility to find the energy which equals a particular E will depend on the portion of total θ_i which corresponds to the same energy E divided by 2π.

We can go one step further. Obviously the above picture of 2-dimensional time has the non-local property: it does not satisfy the causality of 1-dimensional time — we can not link t_1 and t_2 which are on the same circle by the causal relation. The only thing we can expect about causality is the distribution of total time vice space, that is similar to the case in quantum physics, where we can only have the causality for whole wave function. All these arguments can be easily extended to 3- or more- dimension case. Moreover, since we have more degrees of freedom in time, then is that possible to have a rotation in time coordinate which is just the same as spin ?

Although the above picture is rather rough, it still shows the potential what the 2 or 3 dimensional time can realize. First let us guess the dimension of time is 3, just for symmetry reason, later we will find the strong support for this guess. The rest questions are what is the behavior for a free particle which moves in the 3+3 space-time and what is the mechanism in that case? Our job is to find how does the 3-dimensional time maps into 3 dimensional space for a free particle. Is that just the plane wave function? Is it possible that the relations between time and energy and between the position and momentum satisfy the uncertainty relation? Is that possible that the mechanism of the motion of the free electron in 3+3 space-time satisfies the Dirac equation?

Imagine a ghost in microcosm who can watch the single particle without interrupting it and suppose he uses the clock to measure time. He will find when the clock points to time t, this particle shows at lots of different places with different "time angles" t_θ, t_ϕ. Furthermore, he can watch the evolution of the particle in each position when t changes, i.e., he traces the changes of each position(at different t_θ, t_ϕ) when t changes, then there will be lots of paths distributed in space, each path may have different weights, and on each path,the causality is satisfied. This picture is the same as the idea of Feynman path integral. That is, we can let each time path from the center of time sphere to the surface correspond to each Feynman path, and let the surface of time sphere correspond to the surface of wave function. In other aspect, in the whole space the positions of the single particle form a field in which each "force line" passes through each different t_θ, t_ϕ at the specific time, so our question is: how to build a field on a spherical surface S^2 in 3-dimensional time coordinate?

This situation is the same as the situation of Dirac monopole: the "force line" of our field is just the Hopf bundle on S^2. It is well known in monoploe theory [3] that, according to the nontrivial global topological property of S^2, the transition function for monopole bundle on spherical surface is $exp(-2ig\phi)$ (here $\hbar = 1$), and it is not single-valued unless it satisfies the quantization condition:

$$e^{-4\pi ig} = 1. \tag{1}$$

Hence winding number g = 0, $\pm 1/2$, $\pm 1, \cdots$. We will find that, in our case, g is just the spin of the single particle.

Consider a "time" sphere whose radius is 1/2 and sperical coordinate on the surface is (1/2, t_θ, t_ϕ), resting on a $u = x_3 + is$ plane at the south pole. Here x_3 is the 3rd coordinate of space, and s is the combination of space coordinate x and y(each point on s axis corresponds to a circle in x_1-x_2 plane). Let the axis x_0 which corresponds to one specific time direction pass through the south pole o and be perpendicular to x_3-s plane, then the whole coordinate system is shown in Fig1. Let Z= $\begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$ be the stereographic projection coordinates of a point in the northern hemisphere U_n of S^2, where $z_1 = x_0$, $z_2 = z + is.$ on the unit sphere, then we have:

$$x_0^2 - x_1^2 - x_2^2 - x_3^2 = x_0^2 - s^2 - z^2 = |z_1|^2 - |z_2|^2 = 1. \tag{2}$$

Compare it with the condition in Hopf bundle [3] $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1$, the only difference is that the sign of x_i(i=1,2,3) becomes negative. This negative sign comes from the time metric just the same as the time in relativity. Then from (2) and Fig1., we find $x_0 = 1/2(1 + \cosh t_\theta)$, $x_3 = 1/2 \sinh t_\theta \cos t_\phi$, $s = 1/2 \sinh t_\theta \sin t_\phi$, Z can also be written as:

$$Z = \cosh \frac{t_\theta}{2} \begin{pmatrix} \cosh \frac{t_\theta}{2} \\ \sinh \frac{t_\theta}{2} e^{it_\phi} \end{pmatrix}. \tag{3}$$

Since Z is the stereographic projection coordinate, we can omit the common coefficient $\cosh \frac{t_\theta}{2}$, then:

$$Z = \begin{pmatrix} \cosh \frac{t_\theta}{2} \\ \sinh \frac{t_\theta}{2} e^{it_\phi} \end{pmatrix}. \tag{4}$$

Rewrite our representation, let z_1=$\begin{pmatrix} x_0 \\ 0 \end{pmatrix}$, z_2=$\begin{pmatrix} z \\ s \end{pmatrix}$, then the expression of Z can be written as 4-component form:

$$Z = \begin{pmatrix} \cosh \frac{t_\theta}{2} \\ 0 \\ \sinh \frac{t_\theta}{2} cost_\phi \\ \sinh \frac{t_\theta}{2} sint_\phi \end{pmatrix}. \tag{5}$$

If we add the U(1) bundle to the surface, then Z becomes:

$$Z \to Z = \begin{pmatrix} \cosh\frac{t_\theta}{2} \\ 0 \\ \sinh\frac{t_\theta}{2} cost_\phi \\ \sinh\frac{t_\theta}{2} sint_\phi \end{pmatrix} e^{i\chi}. \tag{6}$$

On the north hemisphere, we can let

$$\cosh t_\theta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \tag{7}$$

$$\sinh t_\theta = \frac{\frac{v}{c}}{\sqrt{1-\frac{v^2}{c^2}}} \tag{8}$$

$$cost_\phi = v_3 \tag{9}$$

$$sint_\phi = v_s \tag{10}$$

where $v = \sqrt{v_3^2 + v_s^2}$. Then, we make a substitution:

$$v_s \to v_1 + iv_2 \tag{11}$$

where v_1,v_2,v_3 are components of velocity of free particle in space coordinate, then the Eq. 6 becomes

$$Z = \cosh\frac{t_\theta}{2} \begin{pmatrix} 1 \\ 0 \\ \frac{\sinh\frac{t_\theta}{2} cost_\phi}{\cosh\frac{t_\theta}{2}} \\ \frac{\sinh\frac{t_\theta}{2} sint_\phi}{\cosh\frac{t_\theta}{2}} \end{pmatrix} e^{i\chi} \tag{12}$$

$$= \sqrt{\frac{m + \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}}{2m}} \begin{pmatrix} 1 \\ 0 \\ \frac{mv_3}{m+\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}} \\ \frac{mv_1+imv_2}{m+\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}} \end{pmatrix} e^{i\chi}. \tag{13}$$

If we let m be the static mass of particle, then $mv_i = p_i$(i=1,2,3) and $\frac{m}{\sqrt{1-\frac{v^2}{c^2}}} = E$. Let $\chi = p^\mu x_\mu$ ($\mu =$ 0,1,2,3), we can find that Z is just one of the solution to "positive-energy" of Dirac field in σ_z representation(Here σ is Pauli matrix.) It is easy to see that, let $v_s = v_1 - iv_2$ and interchange the 2nd row and 1st row, and then 4th row and 3rd row, we'll obtain the other solution with "positive-energy". On the south hemisphere, $x_0 = \frac{1-\cosh t_\theta}{2} = \sinh^2\frac{t_\theta}{2}$, and let $\chi \to \chi - t_\phi$, then Eq. 3 becomes:

$$Z = \sinh\frac{t_\theta}{2} \begin{pmatrix} \sinh\frac{t_\theta}{2} e^{-it_\phi} \\ \cosh\frac{t_\theta}{2} \end{pmatrix}. \tag{14}$$

Written as 4-components representation and let $v_s = -v_1 - iv_2$, Z will become the solution of "negative-energy" of Dirac field in σ_z representation. Similarly, if we let $v_s = -v_1 + iv_2$ and interchange the 2nd row and 1st row, 4th row and 3rd row, then we will get the other solution of "negative-energy". So we find that Z in the whole sphere corresponds to 4 solutions of free Dirac field in σ_z representation . Now we put Eq. 6 into Dirac equation

$$(i\gamma^\nu \partial_\nu - m)\psi = 0 \tag{15}$$

and multiply the both sides of the equation by $Z^+\gamma^0$, then we get

$$iZ^+\gamma^0\gamma^\nu\partial_\nu Z = m. \tag{16}$$

Let us make a substitution

$$\partial_{x_1} + \partial_{x_2} \to \partial_s \tag{17}$$

and suppose that in free particle case, t_θ, t_ϕ don't depend on x_i(i=0,1,2,3), then after simple calculation, we have

$$\cosh t_\theta \partial_{x_0}\chi + \sinh t_\theta \cos\phi \partial_{x_3}\chi + \sinh t_\theta \sin\phi \partial_s \chi = -im. \tag{18}$$

However, as we can see from Fig1., the coordinates of unite vector $\hat{n}(n_1, n_2, n_3)$ have the relation that:

$$n_1 = \hat{n} \cdot \hat{x_3} = \sinh t_\theta \cos\phi \tag{19}$$

$$n_2 = \hat{n} \cdot \hat{s} = \sinh t_\theta \sin\phi \tag{20}$$

$$n_3 = \hat{n} \cdot \hat{x_3} = \cosh t_\theta \tag{21}$$

where "·" means the scalar product of two vector. Then the Eq. 18 can be written as :

$$-\hat{n} \cdot \nabla\chi = im \tag{22}$$

or

$$\vec{Q} = -\nabla\chi = im\hat{n} \tag{23}$$

where $\nabla = \hat{x_0}\partial_{x_0} + \hat{x_3}\partial_{x_3} + \hat{s}\partial_s$. Compare the above two equations with the case in static electric field $\vec{E} = -\nabla\phi = \frac{q}{r^2}\hat{r}$, we find that $\vec{Q}$ has the form of classical field and the free Dirac field corresponds to classical static field in 3-dimensional time-space, so we call $\vec{Q}$ "time field".

Here, we use the Dirac field in σ_z representation which corresponds to a particular coordinate system $x_3 - s$ in Fig1. The s axis can be treated as the combination of x_1, x_2, and we make such substitution as Eq. 11 and Eq. 17 since each point on s axis corresponds to a circle in x_1-x_2 plane. In general, the mapping from 3-dimensional space to time spherical surface is completed by 2 steps. First, from the spacial $S^3 \cong SU(2) \to$ spacial S^2, and then from the spacial S^2 to time spherical surface S^2. Instead of Eq. 13, in north hemisphere, Z can be written as:

$$Z = \sqrt{\frac{m + \frac{m}{\sqrt{1-\frac{v^2}{c^2}}}}{2m}} \begin{pmatrix} 1 \\ 0 \\ \frac{m\vec{v}\cdot\vec{\sigma}}{m+\frac{m}{\sqrt{1-\frac{v^2}{c^2}}}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} \end{pmatrix} e^{i\chi}. \tag{24}$$

This is the "positive energy" solution of Dirac equation of free electron in any representation. The same method can be used in the other 3 cases.

Refer to monopole theory. Here Z corresponds to Hopf bundle with winding number g = 1/2 in monople theory. It's easy to see that if we choose the representation Z as the direct product representation of Z_1, Z_2,i.e., $Z = Z_1 Z_2$, where Z_i(i=1,2) are 4-components vectors as eq.(5), then the new Z corresponds to Hopf bundle with winding number g=1. Generally, $Z = \prod_i^n Z_i$ (i=1..n) corresponds to Hopf bundle with g=n/2 . Since each Z_i satisfies eq.(16), the whole Z is just the spinor which satisfies Bargmann-Wigner equation, and the winding number g just correspond to spin in quantum field theory!

From all the above, we can see that spin can be naturally obtained from the topological property of the particle in 3+3 dimensional time-space, but if time dimension is less than or more than 3, we cannot get the same topological result. That is why we think time is 3-dimensional. Also, we obtain 3 interesting observations:

i) The $E > 0$ solution corresponds to the Hopf bundle in north hemisphere, and $E < 0$ solution corresponds to the Hopf bundle in south hemisphere. That means the time direction of electron with negative energy is opposite to the time direction of electron with positive energy, so one can guess that the negative energy corresponds to the electron's past. The reason that the "negative sea" is fully occupied is because the history of electron is fully occupied. So in our case, the Dirac equation is a single particle equation.

ii) If m = 0, we should modify our method to get the field equation. We need move our original point o and the whole $x_3 - s$ plane of Fig1. along x_0 axis 1/2 upward, i.e., move the original point of our coordinate to the center of time sphere, then the whole system has spherical symmetry. This symmetry corresponds to gauge invariant. When $m \neq 0$, we lose spherical symmetry, i.e., we should choose one particular direction as our x_0 axis, and one particular direction as our $\hat{n}$ direction. This case corresponds to non-gauge invariant case. So this mechanism is just the same as the symmetry spontaneous symmetry violation in Higgs mechanism.

iii) In 3+3 dimensional time-space, the most surprised thing is that, even if two particles stay at the same position (x,y,z) and at the same time t, it is possible that they cannot "see" each other, in another word, they have no interaction with each other — if the coordinates of particle 1 is $(t, t_{\theta_1}, t_{\phi_1}, x, y, z)$, but of particle 2 is $(t, t_{\theta_2}, t_{\phi_2}, x, y, z)$. This can interpret why we have the Bose-Einstein Condensate and Superconductivity: when the distributions of time angles of all the particles are fit very well such that, within each small space area, and at time t, no two particles have same time angle t_θ, t_ϕ, then no two particles can "see" each other. These kinds of system will not contain any interaction.

Furthermore, we have found that the solutions of quantum field equation of free particle correspond to Hopf bundles in monopole theory, and each Hopf fiber corresponds to each plane wave with different momentum $\vec{p}$. Consider two extreme cases. When the space position of particle is confined at x at time t — the particle stays at x in all the time angles (t, t_θ, t_ϕ), and each angle in time surface corresponds to one Hope fiber and each Hope fiber corresponds to each different $\vec{p}$, so the different fiber corresponds to each path of particle with different speed. Then after time t, The particle at same t but with different time angles reaches the different positions. This picture conforms to wave-package diffusion in quantum theory; when the particle is in fixed momentum $\vec{p}$, each space point x can only contain one Hopf bundle(i.e., one time angle), and all different time angles will be distributed to the whole space, but with the same Hopf bundle, we can find that particle everywhere. This picture describes the uncertainty relation in 3+3 dimensional time-space.

In addition, we should also add "Wave-Package-Collapse" conjecture to our theory. In our case, that is: if we try to measure some physical attributes of the particle, but by chance, the apparatus only meets a portion of time angle of the particle(suppose the interaction between the apparatus and the particle will change the distribution of time angles of that particle in the whole space.), then after the measurement, the distribution of time angles of the particle will depend on the portion of time angles which we measured. The case is the same as a needle sticks into an inflated balloon, the whole surface of balloon will be destroyed.

The detail of the dynamical properties and the metric of 3+3 time-space will be discussed in future.

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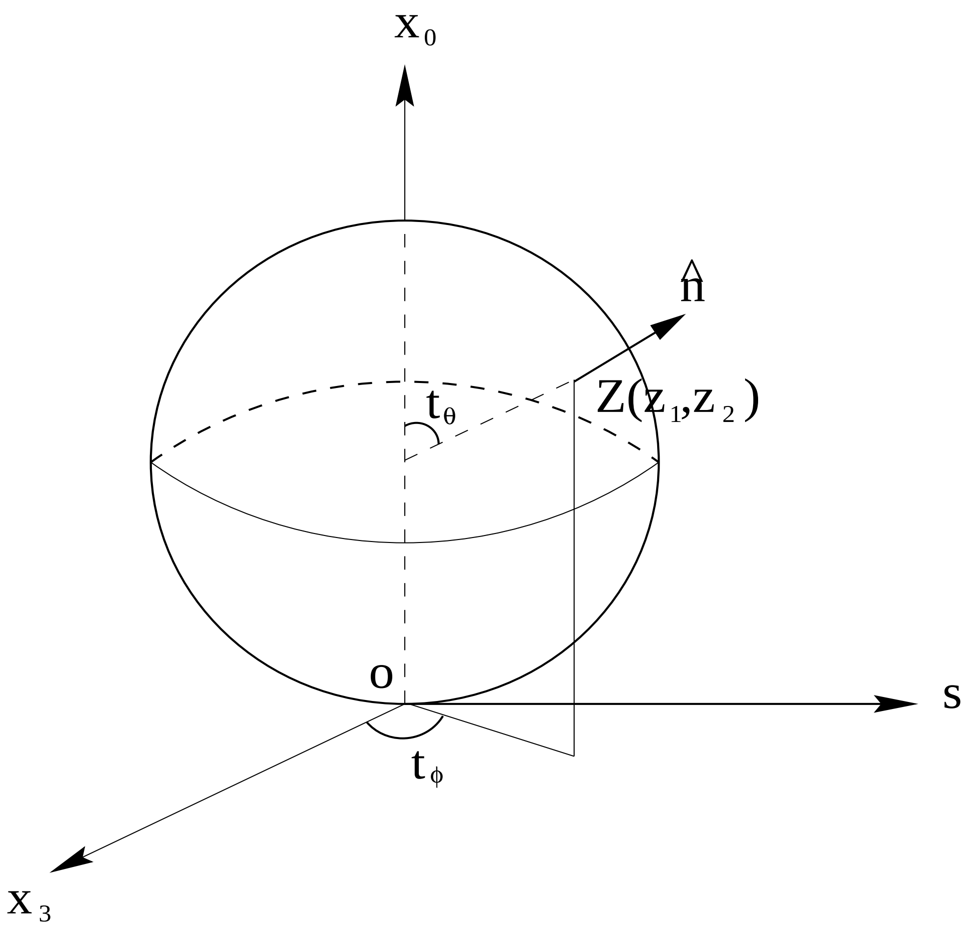

Fig 1.