Einstein’s Hidden Variables:  
Part A - The Elementary Quantum of Light and Quantum Chemistry

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Keywords: Planck, Hidden variable, Light, Time, Photon

Abstract

Re-examination of the work of Max Karl Planck has revealed hidden variables in his famous quantum work, consistent with Einstein’s famous sentiment that quantum mechanics is incomplete due to the existence of “hidden variables”. The recent discovery of these previously hidden variables, which have been missing from the foundational equations of quantum theory for more than one hundred years, has important implications for understanding the interactions of electromagnetic radiation with matter.

Planck’s quantum formula (“E = hν”) is missing the variable for measurement time. Restoration of measurement time produces a more complete quantum formula, “E = hνtm”. Upon balancing, this restored formula reveals that the numerical value of Planck’s constant is actually the mean energy of a single oscillation of light, 6.626 X 10^-34 J/oscillation. Previous definitions of the “photon” relied on a hidden and assumed value for measurement time of “one second”. Quantum chemists of long ago, unaware of this arbitrary value, concluded that the total energy of a “photon” had to be equal to or greater than molecular bond energy, and only “photons” in the visible and ultraviolet regions satisfied this criterion. Infra-red, microwave and radio waves were excluded from study in photochemistry, and were relegated mechanistically to purely thermal processes.

An understanding of the restored quantum formula, with its separate time variable and mean oscillation energy constant, reveals that frequency specific effects range throughout the electromagnetic spectrum and are not limited to just the visible and ultraviolet regions. This awareness, in turn, allows the mechanistic explanation of phenomena related to the use of infrared, microwaves and radio waves in the processing, evolution and performance of materials.

Introduction

There is an elementary quantum of light. It is not the photon. Max Karl Planck glimpsed the elementary quantum of light briefly, but attitudes and beliefs of his time prevented him from seeing it clearly. So for more than one hundred years, the elementary quantum of light has remained hidden in the dusty pages of history.

To understand how this could be, one must look back in time to Berlin in the 1890’s. Planck was a Professor of theoretical physics and many new and exciting discoveries were being made. In 1895, Heinrich Hertz succeeded in transmitting and receiving Maxwell’s “mysterious electromagnetic waves”. Planck embraced the “resonant oscillations” whole-heartedly, and began publishing papers on resonant electromagnetic (EM) waves. He also attempted to prove the irreversibility of entropic processes based on his electromagnetic theory. When Planck presented his first paper in 1897, however, Ludwig Boltzmann loudly criticized his conclusion. Planck had embarrassingly failed to consider certain time dependent effects and had thus not proven the irreversibility of an increase in entropy. He turned to black-body radiation as a way to prove that irreversibility.
Black-body radiation is the light emitted by a theoretical “black-body” or perfect light absorber. A formula to describe changes to the wavelengths of light emitted by an object as its temperature changed was being sought. Black-body radiation devices were the super-colliders of their time, and Planck had ready access to the data generated from the device located in Berlin. The device had an inner chamber lined with the natural black-body material graphite and a second outer chamber which could be filled with either ice or steam. After the graphite chamber reached equilibrium at either 0°C (273 K) or 100°C (373 K), a window in the chamber was opened allowing the emitted black-body radiation to exit and be measured as a function of time. The intensity of various wavelengths could then be obtained to determine the distribution of energy at various wavelengths and at a given temperature.

Among the many equations that had been suggested for black-body radiation, Planck was attracted to Wien’s law, which eliminated time as a variable. He wrote four (4) more papers on black-body radiation and the irreversibility of entropy, using Wien’s law. In those papers Planck developed an early version of his quantum relationship, setting internal energy (“E”) proportional to the product of a constant (“a”), the frequency (“ν”), and the measurement time (“tm”):

\[ E \approx a \nu t_m \]  

(1)

He then used Wien’s method to eliminate the variable for measurement time. By early 1900, Planck used his entropy/electromagnetic theory to publish a derivation and calculation of his famous constant “h” in connection with black-body radiation and his proof of Wien’s law.

In September 1900, however, Planck received a new set of black-body measurements. Wien’s law was wrong. Once again Planck had published a flawed conclusion. He played with the numbers until he found a new equation that fit the data much better than Wien’s. When Planck presented his new black-body equation at a meeting of the German Physical Society the next month, there were no loud critics. His next challenge was to find a proper derivation for his empirical formula, and as Planck later recalled, “The explanation of the... radiation law was not so easy.”

After “some weeks of the most strenuous work of my life”, Planck completed a formal derivation for his new radiation law. He abandoned the wave theory for light, opting instead for a particle-like treatment of light. He also found it necessary to use the statistical approach championed by his nemesis Boltzman, as well as Boltzman’s idea that energy can be divided into small amounts. Planck developed Boltzmann’s energy suggestion into his Quantum Hypothesis, i.e., the idea that energy is quantized in small equal amounts.

Planck’s 1901, formal paper on this topic introduced his famous quantum formula:

\[ E = h \nu \]  

(2)

where Planck’s proportionality constant “h” is equal to 6.626 X 10^{-34} J sec. This fundamental formula is the foundational basis for all of quantum theory. Interestingly, Planck simply assumed this formula and did not derive or prove it. His arbitrary quantum formula yielded a proportionality constant (“h”) equal to the product of energy and time, which Planck referred to as the ultimate “quantum of action”.

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ii The Principle of Least Action, where S is the action (energy • time), and \( S = \Delta E \Delta t \) (e.g., Joule seconds).
Historically, Planck’s paper was a tour de force of nineteenth century physics. He described: 1) the black-body radiation law; 2) the quantum hypothesis; and 3) Planck’s constant “h”. He also calculated two more fundamental constants, “Boltzmann’s constant, k_B”, for the energy of a single molecule at different temperatures, and Avogadro’s number, the number of molecules per mole. Although some in the physics community were slow to comprehend Planck’s monumental achievement, a young Swiss patent clerk quickly grasped the implications of Planck’s incredible feat.

In 1905 Albert Einstein published his remarkable paper on the production and transformation of light, better known as the photoelectric effect. Einstein first noted that “it is quite conceivable...that the [wave] theory of light...leads to contradictions when applied to the phenomena of emission and transformation of light”. He proposed that the interactions of light and matter “appear more readily understood if one assumes that the energy of light is discontinuously distributed in space [in particles]”. Thus the paradox, of broadly spread out waves somehow interacting with small particles of matter, disappeared when both light and matter were thought of as small particles. Einstein then showed a derivation for the mean energy of a single oscillation of an electron, and proposed a mathematical basis for his light packets. Finally, Einstein used his light particle hypothesis to explain various interactions between light and matter including the photoelectric effect and the ionization of gases.

Although Einstein’s work did not meet with immediate acceptance, Arthur Compton’s 1923 paper declaring that “the scattering of X-rays is a quantum phenomenon”, settled the debate in Einstein’s favor. A few years later the term “photon” was coined for these packets of light, and the photon came to be regarded as an elementary particle of nature. Unlike other elementary particles defined by a constant value (such as the electron and its uniform charge) the photon was paradoxically defined by an energy value that is infinitely variable (Fig. 1).

**Figure 1.** Direct relationship between photon energy and frequency, according to $E = h\nu$.

As frequency increases, so too does photon energy. The idea “that light has a very large number of elementary constituents, one for each frequency” – is an oddity that has caused countless hours of consternation for scientists the world over.

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iii $\bar{E} = RT/N$, where $\bar{E}$ = mean energy of electron oscillatory motion, $R$ = universal gas constant, $T$ = absolute temperature, and $N$ = Avogadro’s number. Ibid 7.

iv “The simplest conception is that a light quantum transfers its entire energy to a single electron...” Ibid 7.

v “We have to assume that, in ionization of a gas by ultraviolet light, one energy quantum of light serves to ionize one gas molecule.” Ibid 7.
After Planck and Einstein introduced their quantum concepts many questions and paradoxes arose. For example, experimental observations indicated that light behaved as both a wave and a particle. In 1922, Louis-Victor de Broglie proposed that light waves possess momentum (just like particles), and that particles are “waves” with measurable wavelengths. A few years later, in 1925, Werner Heisenberg developed matrix mechanics for particles, which was the first formal mathematical description for quantum mechanics. The next year, Erwin Schrödinger published his equation on wave mechanics, and showed the equivalence of his wave approach to Heisenberg’s photon-related matrices. A mysterious dimensionless constant - the fine structure constant - was discovered, which defied all explanation.vi

A revolution in quantum mechanics had begun. In 1927, Neils Bohr proposed his Complementarity Principle, postulating that light had a wave-particle duality, and that either a wave aspect of light could be measured, or a particle (photon) aspect could be measured, but not both at the same time. As Einstein later wrote:9

“But what is light really? Is it a wave or a shower of photons? There seems no likelihood for forming a consistent description of the phenomena of light by a choice of only one of the two languages. It seems as though we must use sometimes the one theory and sometimes the other, while at times we may use either. We are faced with a new kind of difficulty. We have two contradictory pictures of reality; separately neither of them fully explains the phenomena of light, but together they do.”

The revolution spilled over into chemistry, and in 1927 Walter Heitler and Fritz London extended Schrödinger’s wave equation for a single electron, to the two electrons in the hydrogen molecule.10 Calculations for bond energies of other molecules soon followed. This new quantum chemistry marked the “genesis of the science of subatomic theoretical chemistry” according to Linus Pauling, and the possibility that chemical heats of reaction could be calculated by quantum mechanics became a seeming reality.11

Paradoxes multiplied like rabbits however. Heisenberg proposed his Uncertainty Principle suggesting that there is always uncertainty in the measurement and determination of any two paired and complementary quantities, such as momentum and position, or energy and time. Bohr and Heisenberg attempted to establish some semblance of order, meeting in Copenhagen in 1927, to develop the “Copenhagen Interpretation” of quantum mechanics. The concepts embodied by the Copenhagen Interpretation later evolved into the Standard Model of Particle Physics, and the paradoxes evolved as well. For example, the Standard Model can explain most forces associated with light and matter, however it cannot explain gravity, a fundamental aspect of our reality. Einstein was able to develop a general relativity theory for gravity, however this resulted in two more contradictory pictures of reality.

Many attempts have been made to unify the fundamental forces of nature, which inherently affect chemical and materials processes. Invariably more paradoxes resulted. Heisenberg’s matrix mechanics can unify gravity and quantum mechanics only with the addition of a mysterious matrix variable. Similarly, quantum gravity theories are plagued by an issue referred to as the “Problem of Time”, i.e., there is a missing time factor. Solutions include a

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vi The fine-structure constant “has been a mystery every since it was discovered more than fifty years ago, and all good theoretical physicists put this number up on their wall and worry about it....It’s one of the greatest damn mysteries of physics: a magic number that comes to us with no understanding by man...” R. Feynman. QED. The Strange Theory of Light and Matter. Princeton Univ. Press, 1988, p. 129
“two-time-physics” which attempts to resolve the Problem of Time by adding another time dimension to the quantum equations.¹²

Both Planck and Einstein were deeply troubled by the paradoxes and uncertainties that their quantum work had spawned. Einstein voiced his concerns formally in his famous “EPR” paper, proposing that quantum mechanics was incomplete because it did not provide a theoretical element corresponding to each element of reality.¹³ He suggested that “hidden variables” were responsible for this incomplete state of affairs. In the 1950’s David Bohm further championed the idea that quantum mechanics is incomplete due to hidden variables, and numerous physicists since then have expressed their similar dissatisfaction with quantum physics.

Recent research has revealed the identities of some of the hidden variables hypothesized by Einstein.¹⁴⁻¹⁶ His formulation for the mean energy of an electron oscillation was extended conceptually, to calculate the mean energy of a single oscillation of light. It was expected that oscillation energy would vary with frequency, just as photon energy does. The startling results, however, were the findings that the mean oscillation energy for light is constant, and numerically equal to Planck’s action constant “h”. The question immediately arose - had Planck’s action constant been misinterpreted so long ago? Was it really an energy constant? If so, it meant a time variable was missing from Planck’s quantum formula.

An extensive foray into the historical records provided answers in the affirmative. Planck’s constant is an energy constant, and not an action constant. As for the missing time variable, it had been present in earlier versions of Planck’s quantum relationship but he omitted it from his black-body derivation. Planck probably did this for what he thought were sound reasons at the time, but which in hindsight, led to needless paradoxes and misinterpretations.

Upon restoration to Planck’s quantum formula, the hidden time variable suggests a far richer interpretation of quantum mechanics and quantum chemistry. Modeling an elementary quantum of light represented by an invariant and universal energy constant – the mean oscillation energy – banishes many of the uncertainties and paradoxes of earlier quantum mechanics.

Derivations and Calculations

§ 1. Derivation of the Mean Energy of a Single Oscillation of Electromagnetic Energy

Start with the mathematical relationships from Planck’s quantum formula for photon energy, “E = hν”, where “ν = N t⁻¹”, and “N” is the total number of oscillations measured per unit time. To obtain the mean energy per oscillation (“É”), divide mean photon energy (“EN”) by the number of waves “N” comprising the photon:

$$\bar{E} = \frac{E_N}{N_{osc}} = \frac{h\nu}{N_{osc}} = \frac{(6.626 \times 10^{-34} \text{ J sec}) (N \text{ sec}^{-1})}{N_{osc}} = 6.626 \times 10^{-34} \text{ J/osc} \quad (3)$$

The mean energy for a single oscillation or wave of light is numerically equal to the value of Planck’s proportionality constant “h” (and can be alternatively represented as “h”).

§ 2. Mean Oscillation Energy is Constant and Independent of Frequency

Consider three different frequencies ν₁, ν₂, such that:

$$\nu_1 << \nu_2 << \nu_3$$
\( \nu_1 = 1 \times 10^3 \) sec\(^{-1}\) (Radio); \( \nu_2 = 2 \times 10^9 \) sec\(^{-1}\) (MW); \( \nu_3 = 3 \times 10^{15} \) sec\(^{-1}\) (UV)  \( (4) \)

Next, obtain the mean photon energy at each frequency \( \nu_{1-3} \) according to Eq. 2 above:

\[
E_1 = 6.626 \times 10^{-31} \text{ J} \\
E_2 = 1.325 \times 10^{-24} \text{ J} \\
E_3 = 1.988 \times 10^{-18} \text{ J}  \quad (5)
\]

The photon energies are all different and are directly proportional to the frequency for each photon (i.e. proportional to the number “\( N \)” oscillations of electromagnetic energy comprising each photon). Next, to determine the mean energy of a single oscillation in each photon, one must divide the mean photon energy by the number of oscillations in that photon:

\[
E_{\text{osc}1} = \frac{6.626 \times 10^{-31} \text{ J}}{1 \times 10^3 \text{ osc}} = 6.626 \times 10^{-34} \text{ J/osc}  \\
E_{\text{osc}2} = \frac{1.325 \times 10^{-24} \text{ J}}{2 \times 10^9 \text{ osc}} = 6.626 \times 10^{-34} \text{ J/osc}  \\
E_{\text{osc}3} = \frac{1.988 \times 10^{-18} \text{ J}}{3 \times 10^{15} \text{ osc}} = 6.626 \times 10^{-34} \text{ J/osc}  \quad (6)
\]

The mean energy of a single wave of light is the same for all three frequencies \( \nu_{1-3} \), namely \( 6.626 \times 10^{-34} \text{ J/osc} \). Performing the same calculations for wavelength, it becomes clear that the mean energy of a single wavelength of light is also \( 6.626 \times 10^{-34} \text{ J} \) for all three different wavelengths \( \lambda_{1-3} \). The mean oscillation energy of light is constant, and does not vary with frequency or wavelength.

\textbf{§ 3. The Photon}

Substituting mean oscillation energy, \( \bar{\hbar} \), for the action constant in Planck’s quantum formula:

\[
\bar{\hbar} \nu = E_N / t  \quad (7)
\]

produces an energy rate. To obtain total energy one must multiply the energy rate by the period of time during which the energy is measured (measurement time, “\( t_m \)”):

\[
E_N = \bar{\hbar} \nu t_m  \quad (8)
\]

This expanded quantum formula can be compared with the traditional condensed quantum formula. Set measurement time equal to a value of “one second”:

\[
E_N = \hbar \nu \cdot 1 \text{ sec} = (6.626 \times 10^{-34} \text{ J}) N = \hbar \nu  \quad (9)
\]

A one second measurement time yields the familiar photon energy value of classical quantum mechanics. Referring to Equation 9, above, it can be seen that:

\[
\hbar = \bar{\hbar} t_m \quad \text{when} \quad t_m = 1 \text{ sec}  \quad (10)
\]

One second time intervals are arbitrary, however, so consider measurement times of differing durations such as three (3.0) seconds or one-half (0.5) second:

\[
E_{N3.0} = \bar{\hbar} \nu \cdot 3.0 \text{ sec} = (1.988 \times 10^{-33} \text{ J}) N  \quad (11)
\]
When measurement time equals some value other than “one second”, the photon energies are different. A longer measurement time measures more oscillations, resulting in greater total energy. When the measurement time equals a numerical value of “one”, the photon energies of traditional quantum mechanics are obtained. The photon, as historically defined, is a time dependent packet of energy, based on the arbitrary measurement time of one second.

An arbitrary one second energy increment cannot be a truly indivisible and elementary particle of nature.

**The Historical Record**

The true brilliance of Planck was in suggesting that energy is quantized in small equal amounts, and in calculating the numerical value - 6.626 x 10\(^{-34}\) J - for that quantum of energy. As Planck commented to Paul Ehrenfest, “Now it seems to me not completely impossible that there is a bridge from this assumption (of the existence of an elementary quantum of electric charge e) to the existence of an elementary quantum of energy”. Planck’s publications leading up to his famous black-body and quantum formula paper, reveal important clues about why he formulated the numerical value of “h” as an action constant rather than an actual energy constant.

In the years just prior to his black-body derivation, Planck published a series of papers on resonant EM radiation processes. In those papers, one finds an earlier version of his quantum relationship:

\[
\begin{align*}
\frac{dS}{dt} \cdot dt &= \frac{I}{a} \frac{dU}{dt} \cdot \log \frac{U}{b} \\
\end{align*}
\]

where “U” is internal energy, and which simplifies to the equivalence:

\[
dU \approx a \nu dt
\]

Clearly Planck was familiar with the mathematical relationship in Eq. 8 above.

Planck adopted the methods of Wien in his electromagnetic work, however, which allowed him to convert the time-dependent energy measurements into energy densities, independent of time. This approach may have been especially appealing to Planck, as it eliminated the source of Boltzman’s earlier criticisms by doing away with time altogether. Of course Wien’s equation and Planck’s electromagnetic work were incorrect, as Planck later admitted in his famous black-body paper:

“... the law of energy distribution in the normal spectrum, first derived by W. Wien from molecular-kinetic considerations and later by me from the theory of electromagnetic radiation, is not valid generally.”

Planck used his flawed theory of electromagnetic radiation, never-the-less, to derive his black-body equation:
‘In any case the theory requires a correction, and I shall attempt in the following to accomplish this on the basis of the theory of electromagnetic radiation which I developed.”

Not surprisingly, Planck found that his black-body derivation required a somewhat arbitrary approach, as well as a certain willingness to overlook simple, practical details:

“You will find many points in the treatment to be presented arbitrary and complicated, but as I said a moment ago I do not want to pay attention to a proof of the necessity and the simple, practical details.”

The preamble to Planck’s formal black-body derivation stated:

“§1. ... we shall refer to the average energy $U$ of a single resonator. Then to the total energy

$$U_N = NU$$

[Planck’s] Equation 1.”

Thus, Planck began his entire black-body derivation assuming that the energy of a vibrating molecule (i.e., a resonator) was independent of time, consistent with the failed Wien equation. His assumption was also contrary to the actual laboratory measurements he used in his derivation, which were quite clearly dependent on time:

“§11. The values of both universal constants $h$ and $k$ may be calculated rather precisely with the aid of available measurement. F. Kurlbaum, designating the total energy radiating into air from 1 sq cm of a black body at temperature $t$ C in 1 sec by $S_t$ found that:

$$S_{100} - S_0 = 0.0731 \text{ watt/cm}^2 = 7.31 \times 10^5 \text{ erg/cm}^2 \text{ sec}$$

(Underlines added)

Ordinarily, one would multiply an energy rate such as this, by the measurement time variable to obtain the total energy. Planck used Wien’s mathematical procedure, however, and instead converted the energy rate to energy density, even though it produced an invalid result:

“From this one can obtain the energy density of the total radiation energy in air at the absolute temperature

$$\frac{4 \cdot 7.31 \times 10^5}{3 \times 10^{10} (373^4 - 273^4)} = 7.061 \times 10^{-15} \text{ erg/cm}^3 \text{ deg}^4$$

Wien’s procedure converted the energy rate to energy density, by dividing the energy rate by the speed of light. This replaced the measurement time variable, with a time constant of “one second” (dividing by the speed of light is the same as multiplying by the constant 1 sec/3 X $10^{10}$ cm). The time variable in Planck’s original quantum relationship thus became fixed at a constant value of “one second”, and the time variable itself was cancelled out and omitted. When Planck derived his constant “$h$”, the product of his original constant “$a$” and the measurement time variable “$dt$”, became condensed into the single constant “$h$”. This resulted in turn, in a
condensed version of his quantum relationship, namely “E = hν”. The variable for measurement time was relegated to an invisible existence as a hidden variable, with an implicit and fixed value of “one second”.

The contradictory assumptions in Planck’s black-body work about energy and time did not go unnoticed by his peers, including Debye:18

“Planck’s Law is verified entirely by experience; and yet the derivation has a weak spot, to the extent that the basic assumptions of both parts on which the proof of the law of radiation is constructed differ from one another. Namely, on the one hand, as is generally known, a connection is established between the average energy of the resonator, using a formulation that is completely determined by its dependence on its momentum and its rate of change [in time] ... But then for the second part of the proof, the pioneering assumption of the existence of elementary energy quanta is made, which nevertheless is in no way connected to the energy description of the resonator which is assumed in the first part; indeed, it virtually contradicts the latter.”

Discussion

Planck’s famous black-body radiation work sparked the entire quantum revolution. It appears, however, that his mathematical assumptions significantly obscured and limited early quantum concepts, impacting the subsequent development of quantum mechanics.

It is tempting to conjecture as to why Planck adopted those particular assumptions. Planck was certainly familiar with the expanded form of his quantum formula that included time as a separate variable, having used that relationship in his earlier papers. Boltzmann’s harsh critique may have predisposed Planck to avoid time altogether in his black-body derivation and his quest to prove the irreversibility of an increase in entropy. A mathematical approach such as Wien’s, that did away with the dependence of energy on time, could have been very appealing.

Planck may have also realized that his original quantum relationship would have defined his constant as energy per oscillation or wave. Significantly, the wave and particle theories for light were thought at that time to be mutually exclusive, and an understanding of wave-particle duality did not yet exist. Planck may have wished to avoid introducing the additional contradictions that would have resulted from his use of wave-related nomenclature in a statistical derivation of particles. Little supposition is required to understand that Planck’s peers would have viewed such a construct as nonsense. It is not surprising therefore that Planck avoided the use of wave or oscillation nomenclature in his quantum formula and determination of “h”, because to do otherwise would have contradicted the very premise upon which his derivation was based.

Unbound by such restrictions, one can now consider concepts that would have seemed quixotic a century ago. If one logically extends Einstein’s model, for the energy of a single electron oscillation, to the energy of a single electromagnetic oscillation, hidden meaning is revealed in Planck’s work. Planck actually found the value for the mean oscillation energy of light. That oscillation energy is constant and does not change with frequency or wavelength. Thus it is conserved over time and space. This suggests that Planck’s energy quantum, invariant under a shift in time or space, is the true elementary particle of light.

Although the photon has long been held to be the elementary quantum of light, that understanding appears to have arisen from limitations in Planck’s abbreviated quantum formula. The variable for measurement time was omitted as a separate independent variable, and was
instead condensed into his action constant “h”, with a fixed hidden value of “one second”. Subsequent use of the condensed quantum formula for calculation of photon energy, inadvertently defined the photon as an increment of light of one second’s duration, regardless of its frequency, wavelength, or energy. The unsatisfying result of this definition was a fundamental particle of light that was infinitely variable.

Einstein voiced his dissatisfaction with quantum mechanics in his epic EPR paper asserting that quantum mechanics “…does not provide a complete description of the physical reality”. He suggested that hidden variables were at the root of the problem. Attempts to test Einstein’s hypothesis – both pro and con - invariably utilized Planck’s incomplete quantum formula. It is axiomatic that an incomplete foundational equation cannot provide for a complete theory. Planck’s complete quantum relationship satisfies Einstein’s desire for an element of reality corresponding to each mathematical element of quantum mechanics.

As noted by David Bohm, “the physical interpretation of the quantum theory centers around the uncertainty principle”. Changes in energy and time are uncertain to the extent that their product must always be greater than or equal to Planck’s constant (\(\Delta E \Delta t \geq h\)). Replacing “h” with “\(\hat{h} \Delta t\)” one discovers far greater certainty in the relationship “\(\Delta E \geq \hat{h}\)”. A change in energy related to light can never be smaller than the energy of a single wave of light.\(^\text{vii}\) Without a separate variable for time in Planck’s quantum formula, it was inevitable that calculations of quantities involving time such as momentum and time itself, would yield uncertain results in the equations of quantum mechanics.

It is likely more than coincidence that both matrix mechanics and two-time physics resolve the quantum/general relativity contradictions by using additional variables. The added nonspecific matrix variable should correspond to the hidden time variable. For two-time physics, the correspondence is self evident. Once again, Einstein’s desire for an element of reality corresponding to each mathematical element can be satisfied.

Similarly, the mystery of the dimensionless fine structure constant becomes far less paradoxical upon use of mean oscillation energy in quantum mechanics. The fine structure constant is no longer dimensionless, and possesses the dimensions of “osc ∙ time”. The product of mean oscillation energy and the fine structure constant then provides the action of light “energy ∙ time”. Both Einstein and Feynman would have been comforted.

In addition, if the elementary particle of light is indeed the single wave of light, the wave-particle duality of light suddenly becomes much less problematic. The fundamental particle of light is a wave. And the single wave of light is light’s fundamental particle. Wave-particle duality seems much less a conundrum when considered from this perspective. The wave and the particle of light are not simply dual, they are identical.

Apart from the rich theoretical implications attached to this re-discovery of Planck’s complete quantum formula, there are many practical applications for this new understanding as well. The development of twentieth century technology was deeply affected by the interpretation assigned to Planck’s condensed quantum formula and constant. The foundations of modern chemistry, for example, were established using comparisons of “photon” energies with molecular bond energies. Re-discovering Planck however, one realizes that the photon energies used by chemists did not necessarily correspond to a realistic element of chemical reality. Molecules are not necessarily restricted in their interactions with light, to intervals of one second’s duration. Freed from the earlier restrictions of an incomplete formulation, it is now

\(^{\text{vii}}\)Since “\(\Delta E \Delta t \geq \hat{h}\)” and “\(h = \hat{h} \Delta t\)”, then substituting we find that “\(\Delta E \Delta t \geq \hat{h} \Delta t\)”. Simplifying we obtain the relationship “\(\Delta E \geq \hat{h}\)”. 582
possible to begin developing more realistic models for the decades of unexplained experimental phenomena related to the production and transformation of light, and the effects of electromagnetic energy and fields.

Numerous experiments in recent decades have demonstrated the frequency-based effects of microwaves and radio waves. According to the teachings of classical photochemistry, such effects were impossible. Neither the microwave nor the radio wave “photon” contained as much energy as a molecular bond. Meanwhile, the microwave and radio spectroscopists were busy irradiating materials with these low frequency waves and observing the resulting frequency-specific excitations, in spite of the “photon” calculations. It now seems apparent in retrospect that the photon calculations were flawed, and imposed unreasonable limitations on concepts regarding matter and the transformation of light. It is reasonable to suggest that a non-thermal interaction between energy and matter depends not on the energy of one second’s duration, but rather on energy of the correct frequency, i.e., a frequency which is sufficiently resonant with the matter, so that it couples with and is absorbed by the matter. This mechanism has been demonstrated countless times in spectroscopy studies and is sufficient to explain the here-to-fore anomalous non-thermal and resonant effects of lower frequency waves on materials.

In addition, it is a well accepted method in low frequency spectroscopy studies to irradiate a material with microwave or radio frequencies, and to detect the frequency-specific excitation of the material by observing its emission of visible and ultraviolet light. The concept that visible and ultraviolet light can catalyze chemical and material reactions and transformations is not controversial. It should not therefore be controversial that processes which stimulate the emission of visible and ultraviolet light can also catalyze chemical and material reactions and transformations. Once again, the resonant frequency effects of microwaves and radio waves on materials lend themselves to reasonable mechanistic explanation.

Finally, it has recently been discovered that materials may be stimulated by low frequency electromagnetic waves, at the resonant acoustic frequencies of the materials. This mechanism has led, for example, to the enhanced growth of biologics using resonant radio waves.

Conclusions

An understanding of the restored quantum formula, with its separate time variable and mean oscillation energy constant, reveals that frequency-specific effects range throughout the electromagnetic spectrum and are not limited to just the visible and ultraviolet regions. This awareness, in turn, allows the mechanistic explanation of phenomena related to the use of infrared, microwaves and radio waves in the processing, evolution and performance of materials.

Acknowledgements

My sincere thanks to Mark Mortenson (General Resonance, LLC); Rustum Roy (Professor Emeritus, Materials Research Laboratory, The Pennsylvania State University); and Hans-Peter Dürr (Professor and Director Emeritus, The Max Planck Institute for Physics, University of Berlin) for their thoughtful comments during this re-examination of Planck’s work.

References

Einstein’s Hidden Variables: Part B - The Resonance Factor

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Keywords: Planck, Boltzmann’s constant, Resonance, Helmholtz energy

Abstract

Re-examination of the work of Max Karl Planck has revealed hidden variables, consistent with Einstein’s famous sentiment that quantum mechanics is incomplete due to the existence of “hidden variables”. The recent discovery of these previously hidden variables, which have been missing from foundational equations for more than one hundred years, has important implications for understanding the interactions of electromagnetic radiation with matter.

Planck tried to integrate the new “resonant Hertzian (electromagnetic) waves”, with existing Helmholtz theories on energy and thermodynamics. In his famous 1901 paper on black-body radiation, Planck described two significant hypotheses – his well known Quantum Hypothesis, and his obscure Resonance Hypothesis. Few scientists today are aware that Planck hypothesized resonant electromagnetic energy as a form of non-thermal energy available to perform work on a molecular basis, bridging the gap between classical Helmholtz thermodynamics of the bulk macrostate, and electrodynamics of the molecular microstate.

Since the black-body experimental data involved only a thermal effect and not a resonant effect, Planck excluded the resonant state in his black-body derivation. He calculated Boltzmann’s constant “k_B” using completely thermal/entropic data, arriving at a value of $1.38 \times 10^{-23}$ J K$^{-1}$ per molecule, representing the internal energy of a molecule under completely thermal conditions. He further hypothesized, however, that if resonant energy was present in a system, the resonant energy would be “free to be converted into work”.

Planck seems to have been caught up in the events of the quantum revolution and never returned to his Resonance Hypothesis. As a result, a mathematical foundation for resonance dynamics was never completed. Boltzmann’s constant was adopted into dynamic theories without its natural companion, the resonance factor (“r_f”). The resonance factor allows the explanation of phenomena related to electromagnetic energy and materials processing, evolution and performance.

Introduction

Boltzmann’s constant is incomplete. It is missing its resonance factor. The Boltzmann constant describes the internal energy of a single atom or molecule, based solely on its temperature, and was described by Max Planck in his famous black-body radiation paper. That work contains Planck’s description of the black-body equation, his Quantum Hypothesis, the quantum formula, $E = h\nu$, Planck’s constant “h”, Avogadro’s number, and Boltzmann’s constant “k_B”. Although Planck was the first to derive and calculate the Boltzmann constant, his heavy reliance on Ludwig Boltzmann’s statistical mechanics resulted in the constant being named after Boltzmann. Many are familiar with these many spectacular aspects of Planck’s landmark work, and especially the fact that it sparked the entire quantum revolution.

Far fewer are familiar, however, with Planck’s Resonance Hypothesis which he proposed in that same paper. In the five (5) years leading up to his black-body radiation work, Planck tried to integrate recent, new data on the “resonant Hertzian (electromagnetic) waves”, with existing theories on energy and thermodynamics. He developed a Resonance Hypothesis in regards to
electromagnetic waves and described it briefly at the beginning of his black-body paper. However, since the black-body experimental data involved only a thermal effect and not a resonant effect, Planck devoted the remainder of his paper to derivations based *solely* on non-resonant, thermal effects. When Planck derived the Boltzmann constant in his famous black-body paper, he excluded the resonant state and calculated $k_B$ without its natural companion, the resonance factor ("r_f"). Subsequent use of the Boltzmann constant without its resonance factor led to the isolation of thermodynamic theory from the realities of many systems in science and industry.

To understand this remarkable state of affairs, one must look back in time beyond Planck, to the 1600’s and the work of Galileo Galilei. Around 1602, Galileo began investigating the pendulum as a device for measuring and keeping time. When Galileo gave small taps to his pendulum, exactly timed to match the frequency of the pendulum swing, its swings grew larger and larger (Fig. 1 below). Galileo had discovered the basis of resonance. Resonance occurs when the frequency of an applied vibration, motion, or energy matches the natural frequency of a system, and the amplitude of the system’s vibrations increase in response.

![Figure 1. Resonance and oscillation amplitude as a function of time.](image)

Galileo discovered this phenomenon by giving small taps to his pendulum, which matched the frequency of the pendulum’s swings. The pendulum oscillations grew larger and larger even though his taps were very small.

At about the same time that Galileo was performing his research with the pendulum, the remarkable French mathematician, Pierre de Fermat, was born. He is best known for his work on number theory, probability, and Fermat’s principle of optics. What is less well remembered about Fermat is that he developed the first equation for resonance. Fermat’s resonance curve demonstrated that as the rate of a mechanical vibration (e.g., a tap) neared the natural vibratory rate of a body (e.g., the swing of a pendulum), the amplitude of vibrations in the body increased.

About twenty years later, famed English scientist, Sir Isaac Newton, published his three laws of motion. From his second law – force equals mass times acceleration - another resonance equation was derived:

$$A = \frac{a}{\sqrt{\nu^2 - \nu_o^2}}$$

(1)

where “$A$” is the amplitude of the system’s oscillations, “$a$” is the acceleration in the system’s oscillation caused by the force of the outside vibrations, “$\nu$” is the resonant frequency of the system, and “$\nu_o$” is the frequency of the outside vibrations that are applied to the system. As the
frequency of the outside vibrations nears the resonant frequency of the system, the denominator in Eq. 1 becomes very small. Even for small accelerations “a” (e.g. small taps), dividing by a very small number produces a large amplitude (Figure 2. below).

![Amplitude plot](image.png)

*Figure 2. Amplitude plotted as a function of an applied resonant force. As the outside frequency “v_o” approaches the resonant frequency “v”, the amplitude of the system’s oscillations increases dramatically.*

In 1748, the Italian mathematical prodigy Maria Gaetana Agnesi described Fermat’s resonance curve in greater detail in her book on integral calculus and differential equations. The book was widely translated throughout Europe and caused such a sensation, that Pope Benedict XIV appointed her to the mathematics chair at the University of Bologna. The idea of resonance and the mathematics to describe it, became widely known and accepted as a scientific principle.

A century later, famed German physician and physicist Hermann von Helmholtz explored concepts of both resonance and energy. As a youth, Helmholtz had a great love of the natural sciences. His father arranged for him to attend the royal medical school, where he would receive free tuition in exchange for service in the army. Upon graduation Helmholtz became a Hussar-Surgeon, and set up a small laboratory in his barracks. There he conducted experiments on muscle metabolism, heat, nerve conduction and electromagnetics, which were published by the newly founded Berlin Physical Society. Helmholtz believed that different energies, such as heat, mechanical, electrical, magnetic and chemical, were the “*definite equivalent of one and the same energy, whatever the mode by which one [energy] is transformed into another*”.

Helmholtz became engaged to be married, but there could be no wedding until he obtained a permanent position. This domestic exigency induced Helmholtz to write his famous 1847 treatise on the conservation of energy. Helmholtz’s treatise provided the first good mathematical description of the conservation of energy. He showed that the many different forms of energy are interconvertible, “…*heat, electricity, magnetism, light, and chemical affinity … from each of these different manifestations of [energy] we can set every other [manifestation] in motion*”. The *Annalen* refused to print Helmholtz’s paper, so he had it printed up privately in a pamphlet, which fortuitously had its desired effect. He was appointed Professor of Physiology at Königsberg, married, and continued his work in medicine and physics.

Helmholtz described his ideas about energy in a simple mathematical form:

\[ U = A + TS \]  

(2)
where “U” is the internal energy of a system, “A” is the energy available for orderly work (now known as “Helmholtz energy”), “T” is temperature (Kelvin) and “S” is the disorderly entropy (or as Helmholtz described it the opposite of work, meaning it was the energy unavailable for work). He conceptualized all energy as involving motion, and differentiated between work and thermal energy based on whether the motion was uniform (work) or random (heat) (Figure 3. below).

Moving to the University of Bonn in 1855, Helmholtz began his investigations on sound and the physics of hearing. In 1862 his book on acoustics was published, and in the section on resonance he explained resonance and sympathetic resonance:

“sympathetic resonance. This phenomenon is always found in those bodies which when once set in motion by any impulse, continue to perform a long series of vibrations before they come to rest. When these bodies are struck gently, but periodically, although each blow may be separately quite insufficient to produce a sensible motion in the vibratory body, yet, provided the periodic time of the gentle blows is precisely the same as the periodic time of the body’s own vibrations, very large and powerful oscillations may result. But if the periodic time of the regular blows is different from the periodic time of the oscillations, the resulting motion will be weak or quite insensible.

Periodic impulses of this kind generally proceed from another body which is already vibrating regularly, and in this case the swings of the latter in the course of a little time, call into action the swings of the former. Under these circumstances we have the process called sympathetic oscillation or sympathetic resonance.”

By the 1870’s Dr. Helmholtz’s brilliant interdisciplinary work had distinguished him to such an extent that he was appointed the Chair of Physics at the University of Berlin (the highest physics position in all of Germany at that time). There he lectured students, including the young Max Planck, on conservation of energy, resonance and other topics. His greatest love remained experimentation however, and one of his conditions for accepting the Physics Chair was the construction of the Physikalisch Technische Reichsanstalt (“PTR”) – the experimental physics institute where the black-body radiation experiments were carried out two (2) decades later.
The Black-Body Radiation Experiments

Black-body radiation is the light emitted by a perfect light absorber (i.e., it is black), and the intensity and wavelength of the light naturally emitted by the black-body material changes as its temperature changes. Black-body radiation devices were the super-colliders of their time, and Planck, who by the 1890’s was a Professor of Physics at the University of Berlin, had ready access to the data generated from the black-body device at the PTR. Black-body devices were, paradoxically, entirely white on the outside, so that they could reflect light. The device had an inner chamber ($c_1$) lined with the natural black-body material graphite and a second outer chamber ($K$) which could be filled with either ice or steam (Figure 3). After the graphite chamber reached equilibrium at either 0°C (273°K) or 100°C (373°K), the shutter ($V$) in the chamber was opened allowing the emitted black-body radiation to exit through the opening ($O_1$) and the aperture ($D$), to be measured by the bolometer ($B$) as a function of time (Fig. 4, below). Scientists at the PTR were gathering data to develop a mathematical description of the changes in the black-body light emissions based on changes in its temperature. Importantly, no light was allowed to enter the inner chamber of the black-body device, so that the experimental data it produced would be based solely on thermal changes in the inner chamber.

Figure 4. Schematic of the black-body device used by Ferdinand Kurlbaum at the PTR, to make black-body measurements. In 1898 Kurlbaum found that the black-body radiation from the chamber at 100 °C was 0.0731 watts/cm² more, than when the chamber was at 0 °C.

Planck’s Calculation of Boltzmann’s Thermodynamic Constant

Planck was very familiar with the fact that the black-body device produced only thermal-based data. He assumed this in his famous black-body work and stated in his introduction:

“the law of energy distribution in the normal spectrum is completely determined when one succeeds in calculating the entropy $S$ ... as a function of its vibrational energy $U$. Since one then obtains, from the relationship $dS/dU = 1/T$, the dependence of the energy $U$ on the temperature $T$ ... [$dU = T dS$].”

Planck’s approach to the black-body problem, was that the energy of the black-body’s radiation depended solely on its temperature and entropy, which, according to Helmholtz meant the motion was completely disordered and random. This understanding, that the black-body data was purely thermal and entropic in nature, allowed Planck to adopt Boltzmann’s statistical
mechanics in his black-body derivation. Boltzmann’s statistical approach assumed that the motion of all elements in the system was completely random and disordered, and that their distribution over various energy states could be calculated on this basis. Planck incorporated these ideas in his preamble:

“§1. Entropy depends on disorder and this disorder, according to my [Planck’s] electromagnetic theory of radiation ... depends on the irregularity with which it constantly changes its amplitude and phase ... If amplitude and phase both remained absolutely constant, which means completely homogeneous vibrations, no entropy could exist and the vibrational energy would have to be completely free to be converted into work.”

Planck had attended the lectures of Helmholtz as a student, and had learned that the total internal energy of any system is the sum of its orderly work energy, and its disorderly, thermal/entropic energy, “U = A + TS”. At one end of the spectrum there could be a totally random disordered state in which the internal “energy U [depends solely] on the temperature... [and] the entire problem is reduced to determining [entropy] S as a function of [energy] U”. In other words, since U = A + TS, and A = 0, then:

\[ U = TS \] (3)

The black-body device was purposely designed to measure only the effects of temperature, and so Planck was describing what was well known at the time. Entropy depends on disorder and by definition, describes the energy unavailable for work. The Helmholtz energy depends on order, and describes the energy that is available for work (Fig. 3 above). In the case of the light emitted by the black-body material, the total energy of the system could be calculated based solely on its temperature because the Helmholtz energy (“A”) was equal to zero. This allowed Planck to calculate the Boltzmann constant “k_B”, for the average energy of a single molecule, based only its temperature.

The Boltzman constant cannot describe orderly work energy, however. It was calculated by assuming that the Helmholtz work energy was equal to zero. If Helmholtz work energy is present in a system, the Boltzmann constant can no longer describe the total internal energy of that system or one of its elements, such as a molecule. It can only describe that portion of the internal energy contributed by the temperature and entropy. Thus the Boltzman constant only allows one to calculate the minimal internal energy of a system or molecule.

To describe the molecule’s total internal energy, the additional energy in the system due to the Helmholtz work energy must be accounted for. Planck’s Resonance Hypothesis laid the foundation for doing just that.

**Planck’s Resonance Hypothesis**

Planck’s contributions to the quantum sciences are well known and his quantum formula (E = hν) sparked the whole quantum revolution. Little attention has been paid, however, to his Resonance Hypothesis – namely, the idea that when resonant electromagnetic energy is absorbed by a molecule, the “energy would have to be completely free to be converted into work”. In other words, light can be converted into Helmholtz work energy. The distinguishing feature that determines whether the light will be converted into actual work energy or into simple heat
energy, is the frequency of its oscillations. If the electromagnetic oscillations are “absolutely constant ...[and] completely homogenous”, they will be “completely free to be converted into work.” Planck thus formalized the concept of “resonant Hertzian waves”, or what we now call electromagnetic energy, as a form of non-thermal energy available for work on a molecular basis. This long neglected concept embodied by Planck’s Resonance Hypothesis, may be even more revolutionary than his quantum hypothesis.

Referring again to Planck’s preamble (§ 1. above), Planck integrated his own recently developed electromagnetic theory, with Helmholtz’s ideas on conservation of energy and the total internal energy of a system. Planck elegantly assimilated Helmholtz’s concepts regarding total internal energy at the bulk (or macrostate) level, with the effects of resonant electromagnetic energy at the molecular (or microstate) level. Planck compared and contrasted total entropy with total resonance, describing in the latter case a system consisting of “completely homogeneous vibrations” in which “no entropy could exist”. In other words, since \( U = A + TS \), and \( S = 0 \), then:

\[
U = A
\]  

Planck described resonant electromagnetic waves as orderly “homogenous vibrations”, and then linked that concept to Helmholtz’s teachings, stating that the “energy would have to be completely free to be converted into work.” This was Planck’s insightful Resonance Hypothesis.

Although Planck’s Quantum Hypothesis garnered all the attention in the past, this may now be the time of Planck’s Resonance Hypothesis. Widespread use of photonic and electromagnetic technologies has led to experimental observations which cannot be explained by simple thermodynamic theory. Planck’s Resonance Hypothesis provides the mechanistic explanation. Since the resonant energy is free to be converted into work, the application of resonant energies can produce effects not typically seen under purely thermodynamic conditions. Indeed the fields of photochemistry and photobiology unknowingly relied on Planck’s Resonance Hypothesis.

Planck’s hypothesis and the effects of resonant energies go far beyond these fields, however. His revolutionary resonance concept has been confirmed in a wide variety of systems and phases – solid, liquid, gas, plasma, biologic, organic, inorganic, electrical, magnetic, chemical, materials, and crystalline – and these effects span the entire electromagnetic spectrum. Results range from accelerated growth of plants and animals, to enhanced chemical catalysis, increased crystal nucleation, virtual thermal effects, and resonant phase changes. As Planck hypothesized, when the “resonant Hertzian waves” induce Helmholtz’s “sympathetic resonance” in a system, the energy is “free to be converted into work” and “very large and powerful oscillations may result”.

**Derivations and Calculations**

§ 1. Derivation of the Resonance Factor

The resonance factor is the ratio of internal energy in a resonant system or element, to the internal energy of its counterpart non-resonant, thermal/entropic system or element.

Applying Planck’s model for completely random motion, all of the internal energy in the system is thermal, \( U_T = TS \). The internal energy per molecule in the thermal system (“\( U_{Tm} \)”) is the product of Boltzmann’s constant and temperature, \( U_{Tm} = k_B T \).
If ordered work is performed on the system, yet it is not “completely homogeneous”, a combination of both ordered resonant energy and disordered thermal energy are present and the internal energy in the resonant system (“$U_r$”) is $U_r = A + TS$. Since $A > 0$, then internal energy per resonant molecule (“$U_{rm}$”) is greater than thermal molecular energy, $U_{rm} > U_{Tm}$, and thus $U_{rm} > k_BT$.

Introduce a variable (“$r_f$”) denoting a resonance factor, so that the total internal energy per molecule in the resonant system “$U_{rm}$” is set equal to, $r_f k_BT$:

$$U_{rm} = r_f k_BT$$  \hspace{1cm} (5)

The ratio of resonant to thermal internal molecular energy is given by:

$$\frac{U_{rm}}{U_{Tm}} = \frac{r_f k_BT}{k_BT} = r_f$$  \hspace{1cm} (6)

Since the internal energies of the molecule and the system are proportional (i.e., by Avogadro’s number and the number of moles) the resonance factor is also equal to the ratio of resonant and thermal system energies.

$$r_f = \frac{U_{rm}}{U_{Tm}} \text{ and } \frac{U_{rm}}{U_{Tm}} = \frac{U_r}{U_T} \text{, therefore } r_f = \frac{U_r}{U_T}$$  \hspace{1cm} (7)

When the molecule is completely entropic and has not absorbed any resonant energy, then $r_f k_BT = k_BT$. The resonance factor equals “one”, and the total internal energy per molecule is equal to its minimal value, i.e., “$k_BT$”. When resonant energy is absorbed by a molecule, however, the resonance factor is greater than one, $r_f > 1$, and the total internal energy per molecule is $U_{rm} = r_f k_BT$.

§ 2. Calculation of the Resonance Factor for a Water/Solute System

The value of the resonance factor - $r_f$ - can be determined from experimental data, based on the amount of work the absorbed resonant energy performs on the system. For example, when water absorbed resonant electromagnetic oscillations, it thereafter dissolved more solute than water that had been kept under purely thermal/entropic conditions, even though the water in both the resonant and thermal systems had identical temperature, volume, pressure, solute and dissolution time:\textsuperscript{17}

<table>
<thead>
<tr>
<th></th>
<th>Resonant System</th>
<th>Thermal System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weight Dissolved (g/100ml NaCl)</td>
<td>26.0</td>
<td>23.8</td>
</tr>
<tr>
<td>Moles Dissolved (NaCl)</td>
<td>4.65</td>
<td>4.25</td>
</tr>
<tr>
<td>Heat of solution$^\text{viii}$ (kJ)</td>
<td>17.5</td>
<td>16.0</td>
</tr>
</tbody>
</table>

The heats of solution in the systems are proportional to their internal energies, thus:

$$r_f = \frac{U_r}{U_T} \approx \frac{17.5 \text{ kJ}}{16.0 \text{ kJ}} = 1.09$$  \hspace{1cm} (8)

$^\text{viii}$ Heat of solution taken as 3.76 kJ/mol for the solute NaCl in liquid water.
The resonance factor “$r_f$” can also be calculated indirectly from the ratios of the products in the systems, whose relative concentrations are proportional to their internal energies:

\[
\begin{align*}
    r_f &= \frac{26.0 \text{ g}}{23.8 \text{ g}} = \frac{4.65 \text{ moles}}{4.25 \text{ moles}} = 1.09 \\
\end{align*}
\]

The resonant system possessed 1.09 times more work energy than the thermal system. The electromagnetic energy was converted into work of dissolution, and more solute was dissolved in the resonant system even though the temperature, volume, pressure and dissolution time were identical in both systems.

§ 3. Calculation of the Total Internal Energy per Water Molecule

The resonance factor calculated above can be used to determine the total internal energy per water molecule. For the water irradiated with the resonant electromagnetic oscillations $U_{rm} = r_f k_B T$ and $r_f = 1.09$, therefore:

\[
U_{rm} = (1.09) (1.38 \times 10^{-23} \text{ J K}^{-1} \text{ per molecule}) (294^\circ \text{ K}) = 442 \times 10^{-23} \text{ J per molecule} \quad (10)
\]

Under purely thermodynamic conditions, the molecular internal energy would be only $405 \times 10^{-23} \text{ J}$. Applying the Helmholtz energy equation to a single molecular element in the system:

\[
U_{rm} - U_{T_m} = (A_m + T S_m) - T S_m = A_m = 37 \times 10^{-23} \text{ J per molecule} \quad (11)
\]

The average resonant Helmholtz energy absorbed by the molecular elements in the resonant system, and converted into work, was $37 \times 10^{-23} \text{ J}$.

§ 4. Calculation of Virtual Thermal Effect Resulting from the Resonant Helmholtz Energy

The virtual or apparent thermal effect resulting from the resonant electromagnetic Helmholtz energy can be calculated. Substitute the internal energy of the resonant molecule, for the internal energy of the thermal molecule and solve for the temperature “$T$”:

\[
T = \frac{U^M}{k_B} = \frac{442 \times 10^{-23} \text{ J per molecule}}{1.38 \times 10^{-23} \text{ J K}^{-1} \text{ per molecule}} = 329^\circ \text{ K} = 46^\circ \text{ C} \quad (12)
\]

The water exposed to the resonant electromagnetic energy behaved as though it was at $46^\circ \text{ C}$, even though it was only $21^\circ \text{ C}$. The Helmholtz energy provided a “virtual” or apparent thermal effect, equivalent to an increase in temperature of $25^\circ \text{ C}$. Without the Helmholtz energy, the water would need to be heated to $46^\circ \text{ C}$, to dissolve the same amount of solute.

Discussion

Planck derived the Boltzmann constant “$k_B$” based on data from an experimental arrangement in which all the energy measured was due to thermal/entropic effects. Planck clearly recognized the dynamic balance between work energy and thermal energy, which was well known to physicists of his time (Fig. 3 above). His introduction and preamble discuss the
differences between the thermal/entropic energy, which by definition, is *not available* to do work, and the resonant electromagnetic energy, which *is available* for work (§ 1. above).

Planck probably did not introduce a resonance factor in his black-body radiation paper because it was un-necessary. The black-body device under study had been specifically designed to exclude all forms of energy other than heat. No light - resonant or otherwise - was allowed to enter the inner chamber containing the black-body material. Thus Planck was confident in his assumption that the system was entirely thermal/entropic in nature.

Although Planck introduced his revolutionary Resonance Hypothesis, knitting together concepts of work and electromagnetic energy at the bulk and molecular levels, the incredible events of the quantum revolution may have overtaken him. It appears that he never revisited his Resonance Hypothesis in later writings. After Einstein popularized the Quantum Hypothesis in his famous papers of 1905, the attention of the world was riveted on its breathtaking implications. If Planck did consider a mathematical resonance factor during his work on resonant electromagnetic radiation, it remained hidden in his working drafts, and has been one of Einstein’s “hidden variables” all these years.

By 1909, Planck had been invited by the President of Columbia University to give a series of lectures on “the fundamental laws which rule in the physics of today, of the most important hypotheses employed, and of the great ideas which have recently forced themselves into the subject.” Notably, Planck spent several lectures discussing aspects of his Quantum Hypothesis, but never mentioned his Resonance Hypothesis. He confined his remarks to the purely thermal/entropic aspects of electromagnetic radiation, much as he did in his black-body paper. In his lectures, Planck focused on describing a mathematical foundation for the irreversibility of an increase in entropy (2nd law of thermodynamics), much as Helmholtz had done for conservation of energy (1st law of thermodynamics).

Planck’s lectures show that his thoughts and concepts had evolved. Planck stated, “To be sure, the view of Helmholtz is not broad enough to include irreversible processes... [T]he method of Helmholtz permits of being carried through consistently...[only] so long as one limits himself to the consideration of reversible processes. ... Reversible processes form only an ideal abstraction...[and] irreversible processes are the only processes occurring in nature.” Planck provided no mathematical foundation for his Resonance Hypothesis, which had been based directly on Helmholtz’s method.

In the absence of a well articulated mathematical foundation for the Resonance Hypothesis, the development of resonance sciences was fragmented at best. Spectroscopy – the study of resonant energy distribution vs. Boltzmann thermal distribution - developed without the benefit of a mathematical foundation for Planck’s Resonance Hypothesis. Resonance concepts were given names which described what they are “not” (rather than what they are), or used thermal terms as proxies. For example, the resonant distribution of molecules over their energy states was described in the negative as a “non-Boltzman distribution”. The increase in internal energy from resonance was designated an elevated “vibrational or apparent temperature”.

Turning more closely now to distributions of energy states, the classical Boltzmann thermal distribution assumes that the motion of all the molecules is *random*, using a Boltzmann weight of “$e^{-E/kT}$”. In resonance, however, a change in state is brought about by the resonant energy, producing a resonant (non-Boltzmann) distribution weight of “$e^{-E/\alpha kT}$”. This resonant weighting produces a bulge in the energy state distribution curve (Figure 4 below).

As the increased internal energy of the molecules is converted to work, the molecules fall to progressively lower energy states. The resonant bulge moves through the energy distribution
curve, the way a bulge moves through a snake. Eventually, all of the resonant Helmholtz energy is converted to work. Absent continued resonant electromagnetic irradiation, the system devolves back to a typical Boltzmann thermal distribution.

The work performed by the resonant energy as the system devolves from a resonant state distribution back to a thermal distribution can shift the equilibrium of the system and produce dramatic changes in its chemical and material dynamics. Equilibria of chemical thermodynamics were described by J. Willard Gibbs in 1902, who described chemical free energy ("G") as a variation of the Helmholtz work energy, namely \( G = A + pV \) ("p" is pressure and "V" is volume). The free energy is converted to work, which in turn transforms the reactants into products, decreasing reactant concentration and increasing product concentration. When the free energy reaches a minimum, the system is in state of dynamic equilibrium. Reactants are changed into product and \textit{vise-versa}, however product concentrations no longer increase.

The point at which a system reaches this dynamic equilibrium can be indicated by an equilibrium constant ("K") which is the ratio of product to reactant concentrations at equilibrium (\( K = \frac{[\text{products}]}{[\text{reactants}]})\). The equilibrium constant is also often formulated as \( K = e^{\frac{-\Delta E}{kT}} \), with \( \Delta E \) being either the Helmholtz or Gibbs chemical free energy (Fig. 5.a). When resonant energy is added to a chemical or material system, the free energy in the system increases and more work is performed on the reactants than in a purely thermal system. The concentration of product increases, as does the resonant equilibrium constant ("K_r") which equals, \( K_r = e^{\frac{-\Delta E}{r k T_r}} \). In other words, the free energy equilibrium curve is shifted to the right (Fig. 5.b, below).
Conclusion

When Planck performed his black-body radiation and quantum work, he limited his derivations to the non-resonant thermodynamic state, excluding resonance in his boundary conditions. Although Planck proposed a Resonance Hypothesis, he did not pursue its development. Inclusion of resonance in dynamic considerations, places the Boltzmann constant in proper perspective as a minimal internal (thermal) energy constant. The product of the resonance factor “rf” and the Boltzmann constant provides for total internal energy. Energy state populations in the resonant state are increased, and dynamic equilibria are shifted. A more complete mathematical foundation has been suggested for the electrodynamics of resonance effects, which may provide a realistic model for many unexplained experimental phenomena related to the frequency-specific effects of electromagnetic energy and fields.

References

17. J. Brooks, M. Mortenson & B. Blum. Controlling chemical reactions by spectral chemistry and spectral conditioning. *U.S. Patent Appl. No. 10/507,660*, 2005. Experimental Procedure – Distilled water (500 ml at 20° C) was placed in each of two 1,000 ml beakers. One beaker was irradiated with resonant electromagnetic frequencies for three (3) hours, while the other beaker was placed in an opaque incubator for three (3) hours. The water in both beakers was 23° C. Sodium chloride (250 g) was added to each beaker and stirred identically. The beakers were placed in a darkened cabinet for twenty (20) hours. Temperature was 21° C (274° K). Salinity and concentration were determined using standard methods.