Causality in quantum teleportation

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Quantum teleportation is a protocol capable of sending an unknown quantum state between two parties (Alice and Bob). It consists of two channels: the quantum one that is a maximally entangled bipartite state, and the classical one – a standard communication channel. It turns out that quantum channel looks as transmitting signal both forward and back in time (e.g. Penrose, 1998). Also it leads to a phenomenon of conditional time travel, which was confirmed experimentally by Laforest et al. in 2003. In our work we examine reversal time process in quantum teleportation with quantum causal analysis, which is a new method giving a formal definition and quantitative measure of causal connection in any bipartite system. We consider a modified protocol of teleportation without an ancillary classical channel. Instead of the unitary transformation, made by Bob after receiving a classical signal from Alice, he measures his particle. We move a moment of Bob's measurement in time and watch how causality between the input state, the outcome of Alice's joint measurement, and Bob's outcome changes. It turns out that Bob's outcome is always the effect relative to the first two values even in the case when it was obtained before the input state for teleportation was prepared. So we obtain time reversal causality, but with cause consisting of absolutely random variable representing Alice's measurement outcome. Therefore we can say that Bob can receive a message from random future. On the other hand, an implementation of causal analysis to time reversal treatment of teleportation, which introduces a proper time frame for teleporting qubit (different from observer's time frame), shows that in this special time frame all the effects appear after corresponding causes. Besides this demonstration of time reversal causality, we have considered teleportation of qubit which is in causal connection with another qubit. As a result the possibility of causality teleportation has been uncovered.

Introduction

Quantum teleportation [1] is a protocol which allows transmitting an unknown quantum state from one spatially separated party (commonly named Alice) to another party (commonly named Bob) without movement of any quantum carriers. To perform this operation Alice and Bob need to share a pair of maximally entangled particles. From the moment of its discovery, entanglement attracts attention by apparent violation of relativity. In the case of teleportation relativity is not violated because Alice and Bob also need a classical channel to complete the protocol. Nevertheless quantum information seems to pass through quantum channel that the entangled pair is. Such suggestion implies the presence of signaling in reverse time considered in Ref. [2] and experimentally tested in Ref. [3].

In this paper we consider the question about causality in quantum teleportation. We use quantum causal analysis [4, 5] – a new method, which proposes formal definitions for terms "cause" and "effect" and also proposes a quantitative measure for strength of causal connection. It helps to validate an implementation of time reversal treatment of teleportation and reveals peculiarities of signaling through reverse time.

We also consider a teleportation of qubit which is causal connection with another qubit. As a result we uncover the possibility of "causality teleportation".

General scheme of quantum teleportation

First, let us describe the general idea of quantum teleportation. Suppose there are two spatially separated parties, commonly named Alice and Bob. One of them (Alice) has a particle *A* in some quantum state $|\psi\rangle$ and wants to transmit this state (but not a particle) to Bob. Quantum teleportation is a protocol which allows Bob to obtain this state on his particle *B*. And with agreement with no cloning theorem, during teleportation particle *A* loses its state. So quantum teleportation is a process of transmitting of quantum state in space without movement of quantum particles.

We will consider the simplest variant of quantum teleportation, where teleporting state is a qubit, that is a superposition of two orthogonal states: $|0\rangle$ and $|1\rangle$. For example, it may be polarization degrees of freedom of photon. For the purposes of convenience we will use modified notation for standard Bell basis vectors:

$$\begin{aligned}
|\Phi^+\rangle &\equiv \frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right) \equiv |\Psi_1\rangle,\\
|\Phi^-\rangle &\equiv \frac{1}{\sqrt{2}}\left(|00\rangle-|11\rangle\right) \equiv |\Psi_2\rangle,\\
|\Psi^+\rangle &\equiv \frac{1}{\sqrt{2}}\left(|01\rangle+|10\rangle\right) \equiv |\Psi_3\rangle,\\
|\Psi^-\rangle &\equiv \frac{1}{\sqrt{2}}\left(|01\rangle-|10\rangle\right) \equiv |\Psi_4\rangle.
\end{aligned} \tag{2.1}$$

Initially Alice has a particle *A* in the state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ ($|\alpha|^2 + |\beta|^2 = 1$). Teleportation is based on usage of maximally entangled two-qubit states, for example Bell state $|\Psi_4\rangle$. One particle from entangled pair goes to Alice (*C*), and another one *B* goes to Bob (Fig. 1a).

In the first step Alice makes a join measurement on particles *A* and *C* and gets some state from Bell basis: $|\Psi_?\rangle$. The question mark is sub index indicates that the result it totally random. In Einstein's terms we can say that it is "a result playing dice of God with the Universe".

Alice's measurement causes a collapse according to identity

$$\begin{aligned}
&\left(\alpha|0\rangle+\beta|1\rangle\right)\otimes|\Psi_4\rangle=\\
&\frac{1}{4}\Big(|\Psi_1\rangle\otimes\left(\alpha|1\rangle-\beta|0\rangle\right)+|\Psi_2\rangle\otimes\left(\beta|0\rangle+\alpha|1\rangle\right)+|\Psi_3\rangle\otimes\left(\alpha|0\rangle-\beta|1\rangle\right)-|\Psi_4\rangle\otimes\left(\alpha|0\rangle+\beta|1\rangle\right)\Big)
\end{aligned} \tag{2.2}$$

The particle *B* turns into one of four pure states, depending on what result Alice has obtained. To get state $|\psi\rangle$ Bob needs to transform his state of *B* but he doesn't know which transformation he needs to apply. But Alice does. She sends the result of her measurement (one of four numbers or 2 bits of classical information) by any classical communication channel. Bob applies proper transformation U and obtains his particle B in state $|\psi\rangle$. In Fig, 1a we use the question mark again to emphasize that this transformation depends on Alice's result.

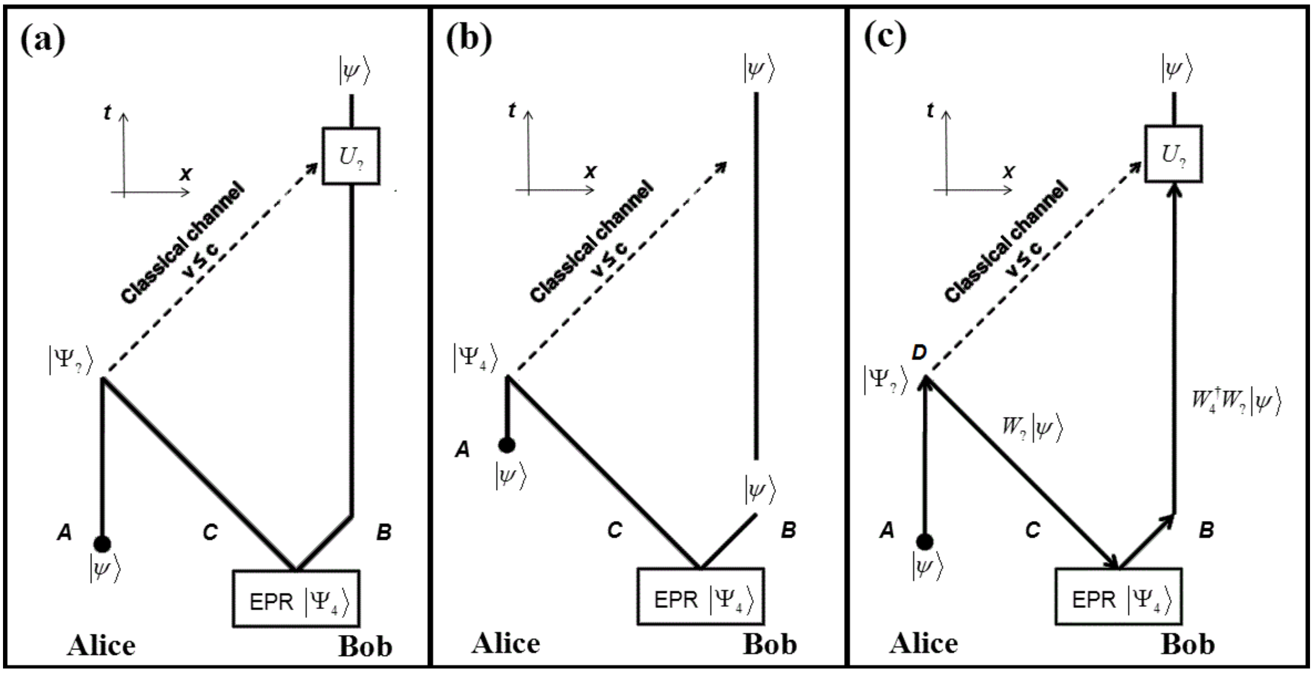

Fig. 1: (a) General scheme of quantum teleportation. (b) Conditional time travel. (c) Time reversal treatment of quantum teleportation.

Conditional time travel and the time reversal treatment

There is an intriguing peculiarity of quantum teleportation, called conditional time travel. If Alice obtains in her measurement the same state as the initial state of *CB* (in our case it is $|\Psi_4\rangle$) then Bob transformation U will be represented by identity matrix. It means that Bob already has his particle in proper state (see Fig. 1b). The question is from which moment Bob already has his particle in proper state. From the viewpoint of standard mathematical approach it seems that Bob's particle collapses in proper state in the moment of Alice's measurement. But it is strange because the problem with instantaneity in space-like interval appears. The last candidate is a moment of EPR pair birth. And really, if we placed a measurement device anywhere on timeline of B, this device would produce statistics like it measures the state $|\psi\rangle$, but only in the case when Alice will get a proper result in her measurement. That is why it is called *conditional* time travel.

In Ref. [3] it was developed an alternative theoretical description for processes in quantum teleportation. The entangled pair has been considered as a channel, which qubit $|\psi\rangle$ follows (see Fig. 1c). Each measurement in basis of entangled states or creation of entangled state has been considered as "time mirror" which changes a direction of qubit propagation in time and makes unitary transformation depending on the corresponding entangled state

$$\left(W_i\right)_{a,b} = \sqrt{2}\left\langle b,a\,|\,\Psi_i\right\rangle. \tag{3.1}$$

So after Bell measurement of Alice qubit $|\psi\rangle$ becomes randomly transformed depending on the Alice's result. Then it goes back in time and becomes transformed once again but this transformation is exact. And then goes forward in time to Bob transformation that appears to equal to inverse of all previous transformation: $U_? = \left(W_4^\dagger W_?\right)^{-1} = W_?^\dagger W_4$.

This new time reversal treatment totally confirms with standard tensor product treatment, but its main feature is that it in intuitive way explains the phenomenon of conditional time travel. Next we are going to consider a question about causality which appears in context of time reversal implementation.

Essence of quantum causal analysis

The standard approach to causality is to suppose that effect is something that goes after cause in time order. But retardation is necessary but not efficient condition for causality and moreover in real situations we often do not measure retardation to know that something is a cause and something is an effect of this cause. It indicates that there is some fundamental asymmetry between cause and effect.

The idea of using information theory to define this asymmetry has resulted in an appearance of causal analysis [6], where the cause is defined as subsystem which influences another subsystem (the effect) stronger than vice versa.

Next let us introduce basic principles of quantum causal analysis [4, 5]. Consider some bipartite quantum system *AB*, which is defined by density matrices ρ_{AB}, $\rho_A = \mathrm{Tr}_B \rho_{AB}$ and $\rho_B = \mathrm{Tr}_A \rho_{AB}$. We can use marginal ($S(X) = -\mathrm{Tr}[\rho_X \log_2 \rho_X]$) and conditional ($S(X|Y) = S(XY) - S(Y)$) von Neumann entropies to construct a pair of so-called independence functions:

$$i_{A|B} = \frac{S(A|B)}{S(A)},\ i_{B|A} = \frac{S(B|A)}{S(B)},\ -1 \le i \le 1, \tag{4.1}$$

which characterize an influence of A on B and B on A.

Causal connection between *A* and *B* corresponds to the inequality $i_{A|B} \ne i_{B|A}$. Then by use of Shannon's theorem about maximal speed of information transmission between *A* and *B* we can obtain minimal times of sending information from *A* to *B* and from *B* to *A*. It turns out that during any period of time effect receives from cause more information than cause receives from effect. Finally we can introduce the velocity of irreversible information flow c_2 (the notation follows the tradition of Ref. [7], where originally, although in less rigorous terms, the course of time pseudoscalar c_2 of the same meaning was introduced):

$$c_2(A,B) = k\frac{\left(1-i_{A|B}\right)\left(1-i_{B|A}\right)}{i_{A|B} - i_{B|A}},\ k = 1. \tag{4.2}$$

Then we can introduce a formal definition for causal connection: A is the cause and B is the effect if $c_2(A,B) > 0$. Absence of causal connection corresponds to $i_{A|B} = i_{B|A}$ and $|c_2(A,B)| \to \infty$, so the less $|c_2(A,B)|$ is the stronger causality is.

The main feature of causal analysis is that it does not use a retardation to define causality. For classical causal connection it can be introduced as an axiom:

$$c_2(A,B) > 0 \Rightarrow \tau_{A\to B} > 0,\ \ c_2(A,B) < 0 \Rightarrow \tau_{A\to B} < 0,\ \ c_2(A,B) \to \infty \Rightarrow \tau_{A\to B} \to 0, \qquad (4.3)$$

where $\tau_{A\to B}$ is time delay between embodiments of A and B.

In Ref. [8] Cramer was the first to distinguish the principles of strong and weak causality. The strong (local) causality corresponds to the usual condition for retardation of the effect relative to the cause described by (4.3). Without this axiom we have the weak causality, which corresponds only to nonlocal correlations and implies a possibility of information transmission in reverse time. We will use the violation of (4.3) in quantum teleportation for revealing of such signals and will see that they can carry only random information (hence "the telegraph to the past" is impossible).

There is one interesting property of c_2 which clearly illustrates its meaning. Consider a set of systems A, B, C, D, E (see Fig. 2) which somehow interact with each other.

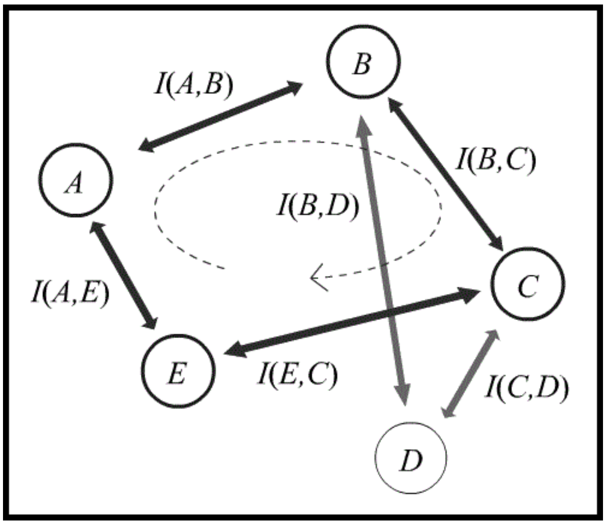

Fig. 2: Illustration of circulation property for c_2.

For any pair of these systems X and Y we can introduce mutual information $I(X,Y) = S(X) + S(Y) - S(XY)$ as a measure of total correlations between them. On the one hand the value of mutual information is symmetric in sense that $I(X,Y) = I(Y,X)$. On the other hand our measure of causality is anti-symmetric: $c_2(X,Y) = -c_2(Y,X)$.

Moreover one can show that (4.2) can be rewritten as

$$c_2(X,Y) = \frac{I(X,Y)}{S(X)-S(Y)}. \tag{4.4}$$

Then, if we choose some closed outline which connects some of these systems and chose a direction in which one can go through this outline we can find that a sum of all values of mutual information divided by corresponding c_2 is equal to zero. E.g. for the outline *A-B-C-E* in Fig. 2 we have $\frac{I(A,B)}{c_2(A,B)}+\frac{I(B,C)}{c_2(B,C)}+\frac{I(C,E)}{c_2(C,E)}+\frac{I(A,E)}{c_2(E,A)}=0$. We can interpret it as the inhibition of causal loops.

Implementation of causal analysis to teleportation

Now we can implement the method of causal analysis to teleportation. First we should consider a standard tensor product treatment. From its point of view teleportation occurs in the moment of Bell measurement. We can consider two configurations of experiment. In case *a* Bob measures his particle *B* before Alice's measurement (Fig. 3a). From the tensor product treatment he just gets some random result. In case *b* Bob measures his particle after Alice's measurement and also gets some random result be this result in encoded version of input state of Alice (Fig. 3b).

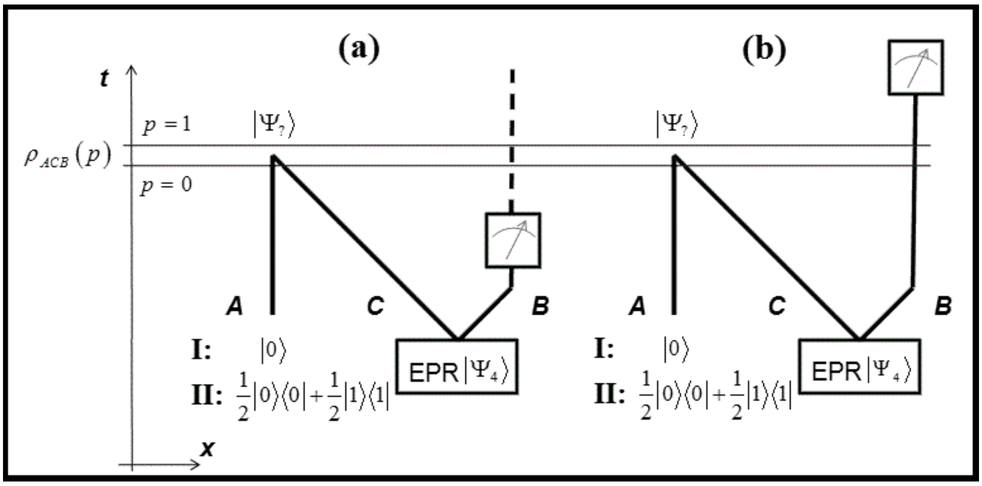

Fig. 3: Two configurations of experiment: (a) Bob measures his particle B before Alice's joint measurement; (b) Bob measures his particle B after Alice's joint measurement.

Also we can introduce two variants of input signal: in variant *I* it is pure state $|0\rangle$ and in variant *II* it is maximally mixed state $\frac{1}{2}|0\rangle\langle 0|+\frac{1}{2}|1\rangle\langle 1|$. So we get four different configurations: *aI*, *aII*, *bI* and *bII.*

To see the behavior of causality we should write a density matrix for the whole system *ACB*. To the purposes of convenience we write it as function of parameter *p*. For $p=0$ we have system just before Bell measurement, for $p=1$ we have system after Bell measurement. Finally we obtain four density matrices during Bell measurement of Alice:

$$
\begin{aligned}
\rho_{ACB}^{aI} &= \rho_{ACB}^{I,\text{out}} + (1-p)\left(\rho_{ACB}^{I,\text{in},\text{mes}B} - \rho_{ACB}^{I,\text{out}}\right),\\
\rho_{ACB}^{bI} &= \rho_{ACB}^{I,\text{out}} + (1-p)\left(\rho_{ACB}^{I,\text{in}} - \rho_{ACB}^{I,\text{out}}\right),\\
\rho_{ACB}^{aII} &= \rho_{ACB}^{II,\text{out}} + (1-p)\left(\rho_{ACB}^{II,\text{in},\text{mes}B} - \rho_{ACB}^{II,\text{out}}\right),\\
\rho_{ACB}^{bII} &= \rho_{ACB}^{II,\text{out}} + (1-p)\left(\rho_{ACB}^{II,\text{in}} - \rho_{ACB}^{II,\text{out}}\right),
\end{aligned}
\tag{5.1}
$$

where

$$
\begin{aligned}
\rho_{ACB}^{I,\text{in}} &= |0\rangle\langle 0| \otimes |\Psi_4\rangle\langle\Psi_4|,\\
\rho_{ACB}^{I,\text{in},\text{mes}B} &= \frac{1}{2}|0\rangle\langle 0| \otimes \left(|01\rangle\langle 01| + |10\rangle\langle 10|\right),\\
\rho_{ACB}^{I,\text{out}} &= \frac{1}{4}\left(|\Psi_1\rangle\langle\Psi_1| + |\Psi_2\rangle\langle\Psi_2|\right) \otimes |1\rangle\langle 1| + \frac{1}{4}\left(|\Psi_3\rangle\langle\Psi_3| + |\Psi_4\rangle\langle\Psi_4|\right) \otimes |0\rangle\langle 0|,\\
\rho_{ACB}^{II,\text{in}} &= \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) \otimes |\Psi_4\rangle\langle\Psi_4|,\\
\rho_{ACB}^{II,\text{in},\text{mes}B} &= \frac{1}{4}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right) \otimes \left(|01\rangle\langle 01| + |10\rangle\langle 10|\right),\\
\rho_{ACB}^{I,\text{out}} &= \frac{1}{8}\left(|\Psi_1\rangle\langle\Psi_1| + |\Psi_2\rangle\langle\Psi_2| + |\Psi_3\rangle\langle\Psi_3| + |\Psi_4\rangle\langle\Psi_4|\right) \otimes \left(|0\rangle\langle 0| + |1\rangle\langle 1|\right).
\end{aligned}
$$

Causality in partition *AC-B* for all situations is presented in Fig.4. We see that causality always amplifies with growth of p and the main peculiarity of causality behavior is that for all four configurations $c_2(AC,B) > 0$ at $0 < p < 1$. This is nontrivial result for cases *aI* and *aII*, where Bob performs his measurement before Alice's one. So form the view point of formal causal analysis it is possible to obtain situation when cause happens after effect.

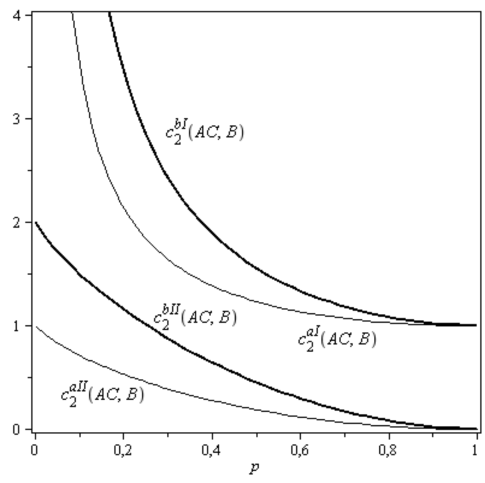

Fig. 4: The behavior of causalities in partition AC-B in different configurations of experiment.

Now let us consider the same four cases with time reversal treatment. It introduces new object *D* which is a result of Bell measurement of *A* and *C*. Moreover in the time reversal treatment there is no differences between the configurations *a* and *b*. Finally we can construct two “density matrices” for the cases *I* and *II* (superscript “tr” emphases that we work in time reversal treatment):

$$\rho_{ACBD}^{I,\mathrm{tr}} = \frac{1}{4}\sum_{j=1}^{4}\left|0,W_j 0,W_4^{\dagger}W_j 0,\Psi_j\right\rangle\left\langle 0,W_j 0,W_4^{\dagger}W_j 0,\Psi_j\right|,$$
$$\rho_{ACBD}^{II,\mathrm{tr}} = \frac{1}{8}\sum_{i=0,1}\sum_{j=1}^{4}\left|i,W_j i,W_4^{\dagger}W_j i,\Psi_j\right\rangle\left\langle i,W_j i,W_4^{\dagger}W_j i,\Psi_j\right|. \tag{5.2}$$

For states (5.2) we obtain the following results: $\left|c_2^{I,\mathrm{tr}}(AC,B)\right| = \infty$, $c_2^{I,\mathrm{tr}}(D,B) = 1$ – these values correspond to the cases *aI* and *bI* at $p = 0$ and $p = 1$; $c_2^{II,\mathrm{tr}}(AC,B) = 1$, $c_2^{II,\mathrm{tr}}(D,B) = 0$ – these values correspond to the cases *aII* at $p = 0$ and *aII* and *bII* at $p = 1$ (see Fig.3). Note that we have obtained $c_2^{bII}(AC,B) = 2$ at $p = 0$, because of $S(CB) = 0$. In time reversal treatment we always have $S(CB) = 1$ because the state “knows” that it will be measured.

In time reversal approach we can consider the new partitions: *AD-C* and *AD-B*. From (5.2) we obtain $c_2^{I,\mathrm{tr}}(AD,C) = c_2^{I,\mathrm{tr}}(AD,B) = 1$ and $c_2^{II,\mathrm{tr}}(AD,C) = c_2^{II,\mathrm{tr}}(AD,B) = \frac{1}{2}$. We see that these values reveal the propagation of qubit through reverse time. In time reversal treatment all the effects appear after corresponding causes (from the view point of formal causal analysis).

Finally we can reconstruct the full picture of causal connections in quantum teleportation. Entangled pair *CB* is a carrier of two signals: the input state *A* and absolutely random result of Bell measurement *D*. Unitary transformation *U* removes influence of random *D* form *B* and Bob gets initial state of *A*. The most interesting is that by measuring *B* Bob doesn’t just get some random result, this randomness comes through reverse time. If we artificially remove randomness from *D* by corresponding postselection we automatically obtain the conditional time travel.

Teleportation of the causal states

Quantum teleportation has one very interesting modification called *entanglement swapping* [9, 10]. Actually it is teleportation of qubit which is entangled with another qubit. After teleportation it appears to be still entangled. In the Fig. 5a we show entanglement swapping between pair *A-C* and pair *A-B* by teleportation of *C* on *B*.

Entanglement swapping is a particular case of more general situation, when *AC* is described by the arbitrary matrix ρ. And after the same operations we will obtain state *AB* in initial state of *AC*. But the state ρ may be causal in sense of informational asymmetry. For example *A* may be a cause with respect to *C* or vice versa (see Fig. 5b).

In such situations we obtain the teleportation of causality. It is the interesting phenomenon, which can take place in the quantum world. It should be noted that teleportation of causality like standard quantum teleportation is limited by speed of light.

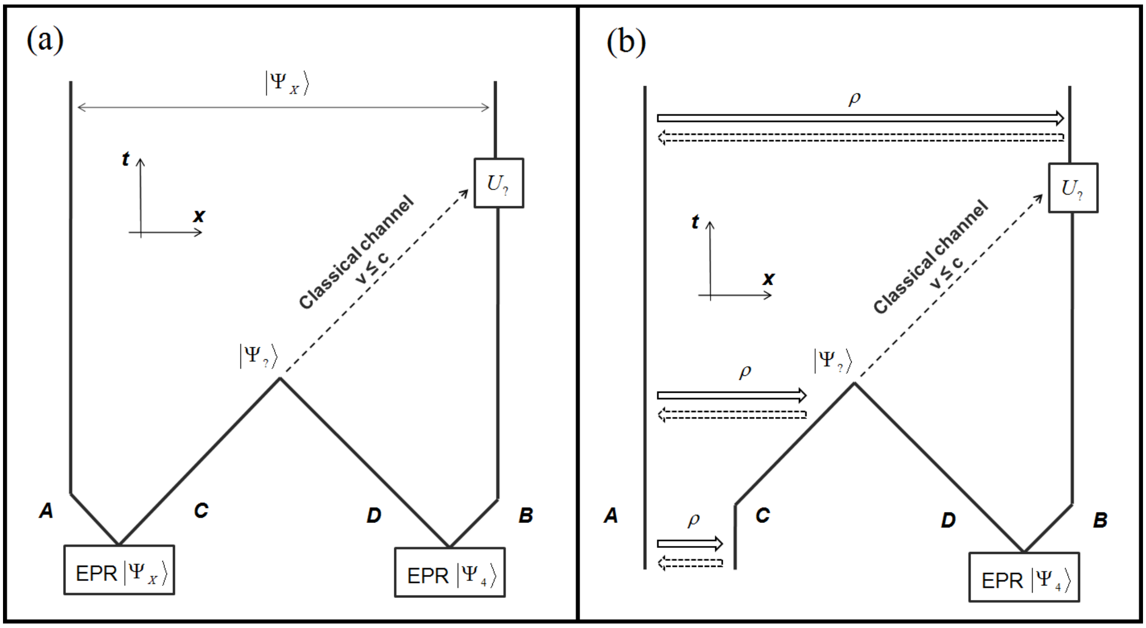

Fig. 5: (a) Scheme of entanglement swapping. (b) Scheme of causality teleportation.

Conclusion

We have considered different treatments of quantum teleportation with quantum causal analysis. Let us make the conclusions.

(1) Causal analysis justifies an implementation of time reversal treatment to teleportation, because exactly in time reversal treatment all the effects appear after the corresponding causes.

(2) Time reversal is an inherent property of quantum entanglement and allows getting information about random future.

(3) Causal analysis shows that "conditional time travel" appears to be a particular case of general signal transmission through reverse time.

(4) Quantum teleportation implies the possibility of causality teleportation, limited by speed of light.

The considered features of time reversal approach may help to understand the experimental results on macroscopic nonlocality (e.g. [11]).

Acknowledgements

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