

Electrodynamics reexamined*

L. S. Shikhobalov

Abstract. It is proved that the electrodynamics laws are the simplest of body interaction laws resulting from the special relativity principles. The result has been obtained with the aid of a model, according which a charged particle is a material point with timelike straight rays going out from it and filling the interior of its the future light cone uniformly. This *radiant model* of a particle corresponds to the Minkowski space geometry completely and does not need Maxwell equations and coordinate systems or systems of reference.

The principles of special relativity are known to be sufficient for the foundation of Maxwell's electrodynamics. However, so far there remains an unanswered question, whether the laws of electrodynamics are unique or, at least, simplest laws of bodies' interaction, satisfying these principles. This work represents one of the steps on the way of solving this problem.

The aim of the paper is *to construct the simplest possible interaction law for point particles of matter within the frames of special relativity without using a priori assumptions about the physical nature of the interaction.*

1 Interaction law

The basic statement of special relativity (in H. Minkowski's interpretation) may be briefly formulated as follows. There exists a four-dimensional manifold (space-time) containing fixed world lines of material points, such that on each world line there is a distinguished direction, coordinated for all the world lines.

Special relativity admits as the space-time manifold the four-dimensional real pseudo-Euclidean space called *Minkowski space*. The distinguished directions on the world lines are called *directions from the past to the future*. The coordination of these directions is expressed in the fact that, for all tangent vectors to the world lines which specify these directions, their scalar products with timelike vectors located within the same cavity of the light cone have the same sign. Note that this condition provides the absence of world lines with spacelike parts.

*ST. PETERSBURG UNIVERSITY MECHANICS BULLETIN (Vestnik Sankt-Peterburgskogo Universiteta. Mekhanika). 1997. Vol. 15, N 3. Allerton Press. © L.S.Shikhobalov

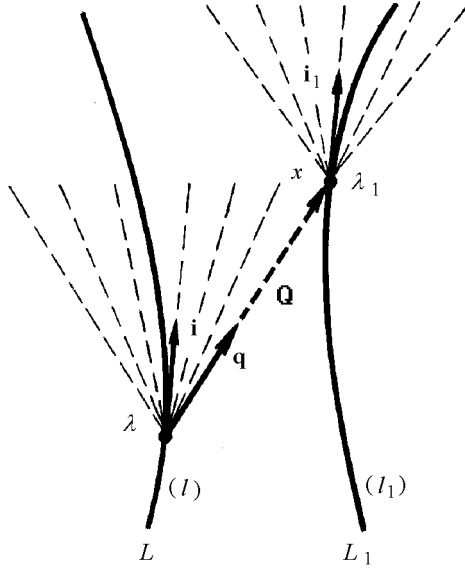


Figure 1: Two particles in Minkowski space and their world lines.

Consider two material points (particles) λ and λ_1 in Minkowski space M , having the *timelike* world lines L and L_1 (see the figure 1). Suppose that other bodies exert no influence upon them. Let us assume that the interaction between the particles λ and λ_1 is a superposition of the influence of the particle λ upon λ_1 and the influence of λ_1 upon λ , obeying the same laws. Let us construct a possible law of such an influence. For certainty, we will speak of the influence of the particle λ upon λ_1 . Let us introduce the notations: \mathbf{i} and \mathbf{i}_1 are the 4-velocities of the particles λ and λ_1 (the unit tangent vectors to L and L_1 directed to the future); l and l_1 are the natural parameters on L and L_1 , increasing to the future.

The influence of the particle λ upon λ_1 determines the shape of the world line L_1 of the particle λ_1 . Let us treat this influence as a certain process that develops successively along L_1 and manifests itself in the changes of the 4-velocity \mathbf{i}_1 of the particle λ_1 as the variable l_1 is increasing. The simplest law of such an influence (linear in the vector $\mathbf{i}_1 dl_1$) has the form

$$d\mathbf{i}_1 = K \cdot \mathbf{i}_1 dl_1 \quad (1.1)$$

where $d\mathbf{i}_1$ is the increment of the vector \mathbf{i}_1 as l_1 changes by dl_1 ; K is a linear operator (a second rank tensor); a dot denotes scalar multiplication. Suppose that the characteristics of the particle λ_1 enter into the expression for K only through a scalar factor. Then the law (1.1) may be rewritten in the form

$$d\mathbf{i}_1 = a_1 F \cdot \mathbf{i}_1 dl_1 \quad (1.2)$$

where $a_1 F = K$; a_1 is the scalar depending only on the characteristics of the particle λ_1 ; F is a second rank tensor which is independent of the properties of the particle λ_1 but

is determined by the properties of the particle λ that exerts the influence, as well as the parameters of the particles' mutual position. Such a situation may be interpreted as the existence of a certain tensor-valued field $F(x)$, $x \in M$, created by the particle λ , which takes (provided the particle world line is fixed) certain values at the points of Minkowski space M and affects any other particle λ_1 placed at a point $x \in M$ according to the law (1.2), where $F = F(x)$.

Let us specify the form of the tensor F .

As $\mathbf{i}_1 \cdot \mathbf{i}_1 = 1$, we have: $\mathbf{i}_1 \cdot d\mathbf{i}_1 = (1/2)d(\mathbf{i}_1 \cdot \mathbf{i}_1) = 0$. Hence, taking into account (1.2), we find: $\mathbf{i}_1 \cdot F \cdot \mathbf{i}_1 = 0$. Since this equality should be valid for any timelike unit vector \mathbf{i}_1 , it is not difficult to prove that the tensor F should necessarily be *antisymmetric*.

The simplest antisymmetric second rank tensor determined by the parameters of the present model (see the figure) and independent of the characteristics of the particle λ_1 is, up to a sign, the tensor

$$\mathbf{q}\mathbf{i} - \mathbf{i}\mathbf{q} \tag{1.3}$$

where \mathbf{q} is the direction vector of the ray connecting the particle λ with the observation point $x \in M$; \mathbf{i} is the 4-velocity of the particle λ ; here and henceforth the tensor product of vectors is denoted without a multiplication symbol between the factors.

Let us adopt some rather natural assumptions.

(a) Suppose that each ray coming from the particle λ gives a contribution to the field $F(x)$ in the form of the tensor (1.3), perhaps multiplied by a numerical coefficient depending on the scalar characteristics of the particle λ and the employed units for measuring physical quantities (the vector \mathbf{q} is specific for each ray).

(b) To satisfy the causality principle, suppose that at each position of the particle λ on its world line it creates the field $F(x)$ only at the points $x \in M$ lying inside its future light cone. In other words, we restrict the admissible vectors \mathbf{q} in the tensor (1.3) to only timelike unit vectors directed to the future (for certainty, we assume that all the vectors \mathbf{q} originate at the particle).

Note that, since in the particle "motion" along its world line, its light cone also "moves" with it, in the case when the world line is infinitely extended to the past, the field $F(x)$ may be in principle created by the particle at any point of the space M . From the mathematical standpoint, there is certainly no obstacle to considering any other vectors \mathbf{q} .

(c) Taking into account the equivalence of all timelike directions, let us suppose that the rays coming from the particle λ are uniformly distributed over all possible directions. Along with the assumption (b) this means that if one builds a pseudosphere of unit radius with a centre at the particle λ , then the ends of the vectors \mathbf{q} will be uniformly distributed over the part of this pseudosphere situated inside the future light cone of the particle.

It is well-known that in real measurements of various physical fields one actually never finds their precise values at separate points at separate time instants, but rather finds their values averaged over certain sufficiently small spatial volumes and time intervals. To reflect this circumstance in our model, let us adopt the following assumption giving the quantity $F(x)$ the meaning of a mean value of the field in a neighbourhood of the point x

(this assumption as though translates the description of the field F from the microscopic level to the macroscopic one).

(d) Suppose that the field F at a point x is determined by an averaged value of the tensor (1.3) over a close neighbourhood of the point x . Hence, due to the mean value theorem, it follows that the contribution to the field F from each position of the particle λ on its world line L is equal to the value of the tensor (1.3) at the point x multiplied by the density of rays near x . Then, if Q is the interval between the particle λ and the point x and $S(Q)$ is the area of the part of the pseudosphere of radius Q centered at the particle, contained inside the particle's future light cone, then, provided the assumption (c) is valid, the density of rays near x is inverse proportional to the area $S(Q)$. (An element of this area is given by the expression $dS = Q^3 \sin \theta \sinh^2 \varphi d\theta d\varphi d\psi$ where θ, φ, ψ are angular coordinates on the pseudosphere.)

(e) Suppose that the contributions to the field F from different positions of the particle λ on L are additive.

On the bases of these assumptions we come to the following expression for the field F created by the particle λ at a point $x \in M \setminus L$:

$$F(x) = a \int_{(L_-)} \frac{\mathbf{q}\mathbf{i}_0 - \mathbf{i}_0\mathbf{q}}{S(Q)} dl. \quad (1.4)$$

Here a is a numerical coefficient, L_- is the part of the world line L of the particle λ contained inside the past light cone of the point x ; $\mathbf{i}_0 = \mathbf{i}$ (later on the vector \mathbf{i}_0 will be given another meaning).

Thus one can conclude that *the simplest interaction law for particles in Minkowski space is specified by the formulae (1.2) and (1.4).*

2 Uniform motion of the particle

In the case of a straight world line L , the tensor F calculated according to Eq. (1.4) exactly coincides (for $a = 4\pi e$) with the electromagnetic field tensor in the situation when the field is created by an electrically charged particle, having the charge e and moving uniformly in vacuum with a velocity corresponding to the world line L . However, in the case of a non-uniform motion of the particle, Eq. (1.4) gives (for $\mathbf{i}_0 = \mathbf{i}$) only one of the two necessary terms in the expression for the electromagnetic field tensor.

Let us improve the model.

3 Arbitrary motion of the particle

Let us supplement the model with the following assumptions.

(f) Suppose that the rays coming from the particle λ are some real objects which are elements of the particle itself. With this assumption taken into account, we will interpret

the word “*particle*” as a material point together with the whole set of straight rays coming from it. The material point itself will be called *centre* of the particle.

To provide the invariable nature of the particle properties with time, let us adopt three more assumptions which state the constant nature of the particle (λ) configuration and its orientation with respect to its world line L in the course of its “motion” along L .

(g) Suppose that the particle, this “hedgehog of rays”, is a non-deformable, absolutely stiff construction, i.e., the intervals between all its points and the angles between all its rays are preserved.

(h) Suppose that in the course of the “motion” of the particle λ along its world line L , the angles between the tangent unit vector \mathbf{i} of the line L and the particle’s constituent rays remain invariable.

(i) Suppose, in addition, that the particle “motion” along the world line L is not accompanied by its rotation around L .

We complete the construction of the model by adopting the assumption which excludes action at a distance from consideration.

(j) Suppose that the influence of the particle λ upon the particle λ_1 is short-range, i.e., is carried out by a direct contact between the rays of the particle λ with the centre of the particle λ_1 . This assumption can be made more concrete in the following way: suppose that the field F is described, as before, by Eq.(1.4), but with the distinction that the quantity \mathbf{i}_0 in this formula is not the 4-velocity of the centre of the particle λ , but the 4-velocity of its point directly interacting with the centre of the particle λ_1 .

We define the 4-velocity \mathbf{i}_0 of an arbitrary point of the particle λ as the derivative of the motion vector of this point with respect to the natural parameter l . Using the assumptions (g)–(i), one can prove that

$$\mathbf{i}_0 = \mathbf{i} + \mathbf{Q} \cdot \left(\mathbf{i} \frac{d\mathbf{i}}{dl} - \frac{d\mathbf{i}}{dl} \mathbf{i} \right) \quad (3.1)$$

where \mathbf{Q} is the vector connecting the centre of the particle λ with its point where the vector \mathbf{i}_0 is calculated (see the figure). In the special case of a straight world line L it holds $\mathbf{i}_0 = \mathbf{i}$ due to $\mathbf{i} = \text{const}$.

A calculation of the field F by the formula (1.4) taking into account (3.1) gives:

$$F(x) = \frac{a(\mathbf{Q}\mathbf{i} - \mathbf{i}\mathbf{Q})}{4\pi(\mathbf{Q} \cdot \mathbf{i})^3} \Big|_* + \frac{a \left[(\mathbf{Q} \cdot \mathbf{i}) \left(\mathbf{Q} \frac{d\mathbf{i}}{dl} - \frac{d\mathbf{i}}{dl} \mathbf{Q} \right) - \left(\mathbf{Q} \cdot \frac{d\mathbf{i}}{dl} \right) (\mathbf{Q}\mathbf{i} - \mathbf{i}\mathbf{Q}) \right]}{4\pi(\mathbf{Q} \cdot \mathbf{i})^3} \Big|_* \quad (3.2)$$

where the asterisk applies to all functions in the expression it marks and means that each function is taken at the point y of intersection between the world line L and the past light cone of the observation point x , for example, $\mathbf{Q}_* = \vec{yx}$ (\mathbf{Q}_* is a null vector). In the case of a straight world line L , the second term in the r.h.s. of (3.2) vanishes. Note that the singularity that appears in the calculation of the integral in (1.4) is compensated by the singularity of the pseudosphere area $S(Q)$.

In order to pass to *observable quantities*, let us introduce some *inertial frame of reference* in Minkowski space M . It is formed by a three-dimensional spacelike hyperplane Γ , called the *physical space*, and a timelike straight line T orthogonal to it, called the *time axis*, with a unit vector τ on T , oriented to the future. Let us use in (3.2) as an independent variable, instead of the parameter l , the coordinate time t counted along the time axis T . Let us form from the tensor F the vector $\mathbf{E} = F \cdot \tau$ and the pseudovector $\bar{H} = (1/2)F \cdot \kappa$ lying in the physical space Γ (κ is the Levi-Civita pseudotensor, the third rank absolutely antisymmetric unit pseudotensor over Γ ; two dots denote biscalar multiplication). One can show that if

$$a = 4\pi e \quad (\text{in SGS system}) \quad \text{or} \quad a = e/\varepsilon_0 \quad (\text{in SI}), \quad (3.3)$$

then the quantities \mathbf{E} and \bar{H} coincide with the electric and magnetic field strengths created by a point particle with electric charge e [1] (these strengths are usually calculated using the Lienard-Wiechert potentials; in (3.3) ε_0 is the electric constant). Bearing in mind that this coincidence takes place for an arbitrary motion of the particle and taking into account that the tensor F is uniquely restored from the quantities \mathbf{E} and \bar{H} using the relation

$$F = \mathbf{E}\tau - \tau\mathbf{E} + \bar{H} \cdot \kappa, \quad (3.4)$$

we conclude that F is the *electromagnetic field tensor*. With such a treatment of the tensor F , under the condition

$$a_1 = \frac{e_1}{m_1 c^2} \quad (3.5)$$

the initial law (1.2) coincides with the well-known *equation of motion of an electric charge in an electromagnetic field* [1] (this equation is usually written in the form $m_1 c d\mathbf{i}_1/dl_1 = e_1 F \cdot \mathbf{i}_1/c$; e_1 and m_1 are the electric charge and mass of the particle λ_1 , c is the velocity of light).

4 Conclusion

Thus we have shown that the simplest particle interaction law within the frames of special relativity is the electromagnetic interaction law. This result has been obtained with the aid of a particle model in the form of a material point with timelike rays coming from it. This *radiant* particle model entirely corresponds to the geometry of Minkowski space and leads in a natural way to the particle interaction law described by the relations (1.2), (1.4) and (3.1). The model does not require the use of the Maxwell equations and the introduction of a frame of reference (the latter is used only for calculating quantities corresponding to the three-dimensional physical space).

The author is greatly thankful to A.D. Alexandrov, A.A. Vakulenko and V.F. Osipov for a detailed discussion of the work and remarks which provided the improvement of a number of statements.

This work was supported in part by the RFBR (grant N 96-01-00431).

References

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