GRAVITY AS A PROJECTION OF THE COSMOLOGICAL FORCE

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The six-dimensional geometrical treatment of gravitation based on the principle of similarity of the basic properties of substance and light is given. To the principle corresponds movement of particles only with the speed of light in a multidimensional space in Compton vicinity of usual three-dimensional space (X) that is subspace of a multidimensional space. The total space is supposed to be six-dimensional Euclidean one (R_6), as for it a simple interpretation of spin of electron and other particles is possible. From the fact of existence of macroscopic three-dimensional bodies it follows, that the particles are kept in a microscopic vicinity of the subspace X by forces (F) of cosmological nature. Particles are moving in R_6 in a geodesic satisfying Fermat's principle, from what the law of conservation of energy of a particle in R_6 follows, and the potential energy appears by the reserved energy of movement in a subspace, which is additional one to the subspace X. The geodesic passes on a pipe surface in R_6 with varying of radius and the speed of light along this pipe. The axis of the pipe is located in subspace X. The curvature of a trajectory is determined by normal component of the force F to the trajectory and the pipe. Metric coefficients with neglect of quantum corrections are determined by unique function of co-ordinates and, in the case of satisfaction of Einstein's equation $R_{00} = 0$, differ from respective coefficients of known spherically symmetric solutions in Einstein's theory of gravitation and the relativistic theory of gravitation only in postpost-Newtonian approximation. The found metrics is obtained as well from a hypothesis about a superposition of local gravitational potentials of partial infinitesimal masses composing a complete gravitational mass. The given treatment is the external geometry of this pipe which is not requires of use of tensor calculation and not supposes of curvatures of space (not space but pipe is deformed) as distinct from the metric theory of gravitation which can be treated as internal geometry of this pipe.

Use of the global principle of simplicity [1] has resulted in six-dimensional geometrical treatment of Lorentz transformations, the interval of relativity, relativistic mechanics, spin and isospin, intrinsic magnetic moment, the fine structure formula, the distinction between particles and antiparticles, de Broglie waves, Klein – Gordon’s equation, CPT symmetry, quark model of particles composed of u- and d-quarks [2,3], and the cosmic expansion [4].

The treatment is based on the principle of similarity of the basic properties of substance and light, examples of that are diffraction of electrons and photoeffect. To it corresponds the assumption that particles of substance move with speed of light in multidimensional space. This assumption goes back to Einstein's statement "the nature saves on principles" and an idea of F. Klein [5-7] that particles move with the speed of light in multidimensional space. It also entered in the principle of simplicity. The first substantiation of six-dimensionality of space was given in [8], where fundamental physical constants are calculated. The six-dimensional treatment of gravitation is given below.

The basic property of light is that in absence of gravitation it propagates with identical speed in any frame of reference. Then the particles of substance should move with the same speed. It is possible only in multidimensional space if positions of particles are recording in projection on homogeneous and isotropic three-dimensional subspace (X). If the formulas of the Newtonian mechanics are referred to six-dimensional Euclidean space R_6, then in projection of events on X these formulas give relativistic results.

The total space is supposed six-dimensional ones, as only for it a simple interpretation of spin of electron and other particles is possible. The particles should be kept in a small vicinity of the three-dimensional Universe by forces orthogonal to it (of a cosmological nature), differently would there are not any macroscopic bodies. We consider a small site of the Universe representing interest at the description of a field of gravitation, as Euclidean subspace X with neglecting of curvature of the Universe on this site. Let us assume that for particles moving in R_6 and considered as material points, formulas of the Newtonian mechanics are applicable at suitable choice of time, which has been mentioned below, and that the positions of particles is fixed by observer in projection on X.

The particle, which is at rest in a projection on X in the inertial frame of reference K of the observer " at rest ", moves with speed of light c in the simplest case in a circle in three-dimensional subspace Y adding up X until R_6, with the center of the circle in X. In any other inertial frame of reference this particle is moving in a helical line located on a cylindrical surface (a motion pipe) in R_6 with an axis in X. We assume that the proper time of a particle is proportional the number of its revolutions in Y around axis of a pipe of movement. This number is proportional to $|\cos\theta|$, where u is the angle of an inclination of a helical line to directrix of the pipe. If the particle makes one revolution per a proper time τ, then by clock of the observer "at rest", relatively which the particle moves along the pipe with a speed $v = c\sin\theta$, it will take place per time $t = \tau / |\cos\theta|$, where

$$\sin\theta = v/c, \qquad \cos\theta = \pm\sqrt{1-(v/c)^2}. \tag{1}$$

In (1) and further positive sign refers to a particle revolving around an axis of a pipe in a positive direction, negative sign concerns to an antiparticle revolving in an opposite direction.

The lapses of proper time of a particle (or antiparticle) $d\tau$ and of time of the observer at rest dt are connected by a ratio

$$dt = \pm d\tau / \cos\theta = d\tau \Big/ \sqrt{1-(v/c)^2}. \tag{2}$$

In the frame of reference at rest K the particle has a component of speed on directrix equal to $c \cdot \cos\theta$. According to (2), the proper time of a particle from the point of view of the observer at rest is proportional to $\cos\theta$ as well, so that the particle in the proper frame of reference K' moves with speed c also.

The displacement of a particle on an interval ds on the directrix of a motion pipe and respective turn on the central angle around of the axis of the pipe, where a is radius of the pipe, are identical in any frame of reference. Having designated through dx in system K a projection of a displacement $d\varsigma$ of a particle on the surface of the pipe on its generatrix and having applied the Pithagorian theorem, one obtains $ds^2 = (cdt)^2 - dx^2$. If to consider this ratio as initial one, then from it follows $d\varsigma = cdt$, i.e. that the particle moves in R_6 with the speed c.

The particle at rest in X is moving in Y with speed c and consequently has a rest momentum $p_y = mc$ and rest energy $E = p_y \cdot c = mc^2$.

By virtue of a principle of similarity of the basic properties of substance and light being a concrete definition of a principle of simplicity, the rest energy mc^2 should also be equal to $h\nu$, where $\textit{н}$ is the frequency of revolution of a particle around an axis of a motion pipe. From here a radius of the pipe equal to $a = \hbar/mc$, and the length of directrix equal to the Compton length that corresponds to the period h of the coordinate of action in the 5-optics [7].

In a field of gravitation the radius a of the motion pipe and speed c_ς of moving over pipe depend on coordinates of subspace X , i. e. from a position of a particle concerning massive bodies. Thus metric coefficients in expression for ds^2 are dependent on a form of functions a and c_ς. The connection between a, c_ς and θ is imposed by a condition, what the particle moves over the pipe in a geodesic according to Fermat's principle. By definition $c_\varsigma = d\varsigma / dt$, where dm is a trajectory segment, which the particle passes over pipe in a time dt by clock of a distant observer. Father, c_ς and a are supposed not depending from the angle θ. Let's show that along a geodesic onto the pipe

$$(a/c_\varsigma)\cos\theta = const. \tag{3}$$

At $c_\varsigma = const$ the geodesic is describing by Clairaut's law $a\cos\theta = const$ [9], and at $a = const$ by Snell's law. In a general case

$$\frac{d\theta}{d\varsigma} = \frac{\partial\theta}{\partial a}\frac{da}{d\varsigma} + \frac{\partial\theta}{\partial c_\varsigma}\frac{dc_\varsigma}{d\varsigma} \quad . \tag{4}$$

From Clairaut's and Snell's laws, by differentiation, one finds $\frac{\partial\theta}{\partial a} = \frac{1}{a}\cot\theta$, $\frac{\partial\theta}{\partial c_\varsigma} = -\frac{1}{c_\varsigma}\cot\theta$, respectively. Inserting these expressions into (4) results in $\frac{d\theta}{d\varsigma} = \cot\theta \frac{c_\varsigma}{a}\frac{d}{d\varsigma}\left(\frac{a}{c_\varsigma}\right)$, which on integration gives equation (3).

The given treatment of gravitation turns out by external geometry of a motion pipe of a particle, while the metric theory of gravitation by internal geometry of the motion pipe.

It should be noted that in each normal cross-section of a motion pipe all radial directions, being orthogonal to the subspace X, are equal in rights even in a case of curved pipe axis. Hence the metrics on a pipe surface does not depend on polar angular coordinate in any normal section, and the internal geometry [9] on a pipe surface is the same as on a respective surface of rotation in three-dimensional space.

The projection of speed c_ς on tangent to a meridian is equal $v_\varsigma = c_\varsigma \sin\theta$. The coordinate speed v of the particle registered by the removed observer equals

$$v = \frac{d\sigma}{dt} = v_\varsigma \frac{d\sigma}{d\xi} = c_\varsigma \sin\theta \frac{d\sigma}{d\xi} = c_\varsigma \sin\theta\left[1 + (\nabla a \cdot \cos\beta)^2\right]^{-1/2} \tag{5}$$

where ξ and y are lengths of arches along a meridian and axis of the pipe, respectively, $в$ is an angle between ∇a and tangent to an axis. Co-ordinate speed of light one obtains by means of tending in (5) θ to $\pi / 2$:

$$c_k = c_\varsigma\, d\sigma/d\xi = c_\varsigma \left[1 + (\nabla a \cdot \cos\beta)^2\right]^{-1/2} \tag{6}$$

On a displacement through distance $d\varsigma$ on the pipe a particle rotates about pipe axis through the angle $d\alpha = d\eta/a$ where $d\eta = \cos\theta d\varsigma$ is projection of this displacement on the directrix of the pipe. The angle $d\alpha$ is the same for any observer, i.e. is invariant, as number of revolutions around of the axis of the pipe is identical for any observer. The quantity $ds = a_\infty d\alpha$, where a_∞ is the radius of the pipe at infinite distance from the center of gravitation, also is invariant. It is an interval of the metric theory of gravitation. Under Pithagorian theorem one has $d\eta^2 = d\varsigma^2 - d\xi^2$. Substituting here $d\varsigma = c_\varsigma dt$, multiplying both parts of the equality on $(a_\infty/a)^2$ and taking into account that $d\alpha = d\eta/a$, one can find

$$(a_\infty d\alpha)^2 = ds^2 = \left(\frac{c_\varsigma a_\infty}{a} dt\right)^2 - \left(\frac{a_\infty}{a} d\xi\right)^2 . \qquad (7)$$

This with account of (6) it is possible rewrite as

$$ds^2 = \gamma (cdt)^2 - \gamma [(c/c_k) d\sigma]^2 , \qquad (8)$$

where c is the limiting value of the speed c_ς on infinity,

$$\gamma = (c_\varsigma a_\infty / ca)^2 . \qquad (9)$$

It follows from (8) that the proper time τ of a particle is connected with time t of an observer removed at infinity, by the relation

$$d\tau/dt = \sqrt{\gamma} , \qquad (10)$$

and the elements of spatial distances dl and $d\sigma$, relatively for local and distant observers, by the relation

$$dl = \sqrt{\gamma} (c/c_k) d\sigma , \qquad (11)$$

and for the local observer

$$ds^2 = (cd\tau)^2 - dl^2 . \qquad (12)$$

The relations (10) and (11) can be obtained as well so. For the local observer the radius of pipe a (or equal to it the Compton length $2\pi a$, which can be measured) stands duty as a scale of length. As a result the lengths are measuring along the meridian, and consequently

$$dl = \frac{a_\infty}{a} d\xi = \frac{a_\infty}{a} \frac{d\xi}{d\sigma} d\sigma = \frac{a_\infty c_\varsigma}{ac_k} d\sigma . \qquad (13)$$

Whence taking into account a designation (9), one obtains the relation (11). A period of revolution in Y of a particle situated near this observer stands duty as a scale of time for the local observer. This period is proportional to c_ς / a, whence formula (10) follows. The speed v_{loc} of a particle for the local observer according to (9), (10) and (13) is equal to $v_{loc} = \frac{dl}{d\tau} = \frac{a_\infty}{a} \frac{d\xi}{dt} \frac{dt}{d\tau} = \frac{a_\infty}{a} c_\varsigma \sin\theta \frac{1}{\sqrt{\gamma}} = c \sin\theta$. By this

$$v/c_k = v_\varsigma / c_\varsigma = v_{loc}/c = \sin\theta . \qquad (14)$$

Whence it is seen that upper limit of local speed for the local observer (at $\sin\theta \to 1$) is equal to speed of light at infinity. The formulas (3) and (10) in view of a designation (9) can be presented as

$$\left(1/\sqrt{\gamma}\right) \cos\theta = const., \quad \left(dt/d\tau\right) \cos\theta = const. \qquad (15)$$

The relations (14) and (15) allow to express speed of a particle through γ:

$$\left(\frac{v}{c_k}\right)^2=\left(\frac{v_m}{c_m}\right)^2=\left(\frac{v_{loc}}{c}\right)^2=1-\frac{\gamma}{\gamma_0}\cos^2\theta_0=1-\frac{\gamma}{\gamma_0}\left[1-\left(\frac{v_{loc}}{c}\right)_0^2\right], \tag{16}$$

where zero in the index marks the values at the initial moment of time.

As the proper time of a particle is measured by number of its revolutions around an axis of a motion pipe, the difference of clock- readings in the end and in the beginning of a journey of an arbitrarily moving observer is proportional to the integral from the interval. In fact, (12) can be represented according to (14) as

$ds^2=(cd\tau)^2\left[1-(v_{loc}/c)^2\right]=(\cos\theta\cdot cd\tau)^2=(cd\tau')^2$, where $d\tau'=\cos\theta\cdot d\tau$ is an increasing in the proper time for this observer. Whence integrating $ds=cd\tau'$ along the trajectory between points A and B, one finds $\tau'_B-\tau'_A=\frac{1}{c}\int_A^B ds$.

For the local observer, the acceleration of a particle is equal to $dv_{loc}/d\tau$. Taking into account (16), one finds $\frac{dv_{loc}}{d\tau}=\frac{dv_{loc}}{dl}\frac{dl}{d\tau}=\frac{1}{2}\frac{dv_{loc}^2}{dl}=-\frac{c^2}{2}\frac{\cos^2\theta_0}{\gamma_0}\frac{d\gamma}{dl}$. Whence the acceleration of gravity force for the local observer will be $g_{loc}=\frac{c^2}{2\gamma}\frac{d\gamma}{dl_{\parallel}}$, where $dl_{\parallel}$ is an element of spatial distance in the direction of gradient of function γ from the point of view of the local observer. Introducing a gravitational potential Φ_{loc} by equality $g_{loc}=d\Phi_{loc}/dl_{\parallel}$ and integrating, one finds

$$\sqrt{\gamma}=\exp\left[-\frac{1}{c^2}\int_{l_{\parallel}}^{\infty}g_{\Lambda}dl_{\parallel}\right]=\exp\left(\frac{1}{c^2}\Phi_{\Lambda}\right)=\frac{d\tau}{dt}. \tag{17}$$

The formula (17) describes a slow down of time in a field of gravitation. Elimination of $\sqrt{\gamma}$ between (15) and (17) and tacking into account (14) one gets that along a geodesic

$$\left[1-(v_A/c)^2\right]\exp\left[(2/c^2)\Phi_{\Lambda}\right]=const. \tag{18}$$

In the domain of weak fields the formulas (17) and (18) are reduced to forms $d\tau/dt=1+(\Phi_{\Lambda}/c^2)$, $(v_{\Lambda}^2/2)-\Phi_{\Lambda}=const.$ Last formula expresses the law of conservation of energy in the mechanics of Newton. Similarly we shall find magnitude of acceleration from the point of view of the removed observer: $\frac{dv}{d\sigma}=\left\{\frac{1}{2}\left(\frac{v}{c_k}\right)^2\frac{d}{d\sigma_{\parallel}}c_k^2-\left(\frac{c_k}{c_{\parallel}}\right)^2\left[1-\left(\frac{v}{c_k}\right)^2\right]g\right\}\cos\beta$,

where $g=\frac{c_{\parallel}^{\;2}}{2\gamma}\frac{d\gamma}{d\sigma_{\parallel}}$ is the acceleration of force of gravity, $d/d\sigma_{\parallel}$ means differentiation in a direction of a gradient of γ, $c_{\parallel}$ is value of c_k in this direction.

The particle at rest in X revolves in Y with frequency $\nu_0=c_{\varsigma}/(2\pi a)$, having energy at rest $E_0=h\nu_0=\hbar c_{\varsigma}/a=\hbar\sqrt{\gamma}\,c/a_{\infty}=m_{\infty}c^2\sqrt{\gamma}$. For a moving particle the total energy will be equal to $E=E_0/|\cos\theta|$. The Lagrangian's formalism yields also the same results.

The action S is determined within accuracy up to a constant factor as integral from a scalar. Scalar here is the angle of turn of a particle around of an axis of motion pipe. A constant multiplier one chooses by such, that in absence of gravitation Lagrange function to be $L = -mc^2 \cos u$, as in the relativistic mechanics. Then one has $S = -\hbar\int_{\alpha_1}^{\alpha_2} d\alpha$. The Lagrange function L is defined by the formula $S = \int_{t_1}^{t_2} Ldt$. From here, one finds $L = -\hbar\dot{\alpha}$. From (7) one has $\dot{\alpha} = (1/a)\sqrt{c_\varsigma^2 - v_\varsigma^2}$, so that $L = -(\hbar/a)\sqrt{c_\varsigma^2 - v_\varsigma^2} = -(\hbar c/a_\infty)\sqrt{\gamma}\cos\theta$. Thus a projection of the momentum of a particle on a meridian of pipe is $p_\xi = \dfrac{\partial L}{\partial v_\varsigma} = \dfrac{\hbar v_\varsigma}{a\sqrt{c_\varsigma^2 - v_\varsigma^2}} = \dfrac{\hbar}{a}\tan\theta$, and energy equals $E = p_\xi v_\varsigma - L = \dfrac{\hbar c_\varsigma}{a\cdot\cos\theta} = \dfrac{\hbar c}{a_\infty}\dfrac{\sqrt{\gamma}}{\cos\theta} = \dfrac{m_\infty c^2\sqrt{\gamma}}{\cos\theta}$, so $p_\xi = Ev_\varsigma/c_\varsigma^2$ and the total momentum of a particle $p = \partial L/\partial c_\varsigma$ becomes $p = E/c_\varsigma = \hbar/(a\cdot\cos\theta)$. From here and (15) it is seen that at moving along a geodesic $E = const$. Thus occurs only flow of energy of movement from the latent form in subspace Y into the evident form in subspace X or on the contrary. The potential energy is the reserved energy of motion in extra dimension space Y.

In absence of gravitation a particle at rest in X revolves in Y along a circumference of radius a_∞ with speed of light c. Appropriate to such revolution centripetal force is equal to $F = p_y c/a_\infty = \hbar c/a_\infty^2 = m_\infty c^2/a_\infty$, in c^2/ag times exceeding weight of the particle at the terrestrial surface, in $2.38\cdot10^{28}$ times for electron. This force may have only cosmological nature. The same result turns out and at movement of a particle on a helical line: $F = pcK/\cos\theta$, where $K = \cos^2\theta/a_\infty$ is the curvature of helical line. From this it is seen that $F = \hbar c/a_\infty^2$ at any θ.

In a field of gravitation the angle of an inclination of a meridian to pipe axis is determined by relations: $\sin\chi = da/d\xi$, $\cos\chi = \sqrt{1-(da/d\xi)^2} = 1/\sqrt{1+(da/d\sigma)^2}$ $\tan\chi = da/d\sigma$. A component of the cosmological force perpendicular to the geodesic, in the osculating plane, is equal to the centripetal force proportional to the curvature K of a trajectory:

$$pc_\varsigma K/\cos\theta = F\cos\chi, \tag{19}$$

where $K = \sqrt{K_\perp^2 + (\sigma'')^2}$, $K_\perp^2 = (y_1'')^2 + (y_2'')^2$; y_1 and y_2 are coordinates of the particle in two mutually perpendicular directions in a section of the pipe, the prime means derivative along a trajectory. It is possible to write these coordinates as $y_1 = a\cdot\cos\alpha$, $y_2 = a\cdot\sin\alpha$. Then, taking into account that $ds = ad\alpha = \cos\theta\, d\varsigma$, $d\xi = \sin\theta\, d\varsigma$, (15) and $\sigma' = \cos\chi\sin\theta$, one finds:

$$K_\perp^2 = (a\alpha'^2 - a'')^2 + (a\alpha'' + 2a'\alpha')^2 = \left(1-\gamma\frac{\cos^2\theta_0}{\gamma_0}\right)\frac{\cos^2\theta_0}{\gamma_0}\left[\frac{d\sqrt{\gamma}}{d\xi} + \frac{\sqrt{\gamma}}{a}\frac{da}{d\xi}\right]^2 +$$
$$+\left[\frac{\cos^2\theta_0}{\gamma_0}\sqrt{\gamma}\left(\frac{\sqrt{\gamma}}{a} + \frac{d\gamma}{d\xi}\frac{da}{d\xi}\right) - \left(1-\gamma\frac{\cos^2\theta_0}{\gamma_0}\right)\frac{d^2a}{d\xi^2}\right]^2, \tag{20}$$

$\sigma'' = -\cos\chi \dfrac{\cos^2\theta_0}{2\gamma_0}\dfrac{d\gamma}{d\xi} - \dfrac{1}{\cos\chi}\left(1 - \dfrac{\cos^2\theta_0}{\gamma_0}\gamma\right)\dfrac{da}{d\xi}\dfrac{d^2a}{d\xi^2}$. Substituting the found expressions for p and F in (19) yields: $a_\infty\sqrt{\gamma}\,K/\cos^2\theta = \cos\chi$. Whence and from (20) to an accuracy of $a_\infty^2\left[\left(d\sqrt{\gamma}/d\xi\right)^2 + \left|d^2\sqrt{\gamma}/d\xi^2\right|\right]$ one obtains $\sqrt{\gamma}\,a_\infty/a \approx 1$. Thus on the basis of (9) one has:

$$a/a_\infty = \sqrt{\gamma}\,, \quad c_\varsigma/c = c_\perp/c = \gamma\,, \tag{21}$$

where $c_\perp$ is the speed of light in a direction, perpendicular to the gradient of the field.

The formulae (21) are follow as well from the equality $E_0 = Fa$ .It means that an increase of the rest energy is equal to the work against the cosmological force: $dE_0 = Fda$.

In the metric theory of gravitation, it is considered that the field of gravitation is generated only by massive bodies and is accompanied by decrease of speed of light and by a slowing down of proper time in vicinity of massive bodies. In six-dimensional treatment of gravitation, massive bodies themselves gravitation does not create, they only decrease speed of light in bodies' vicinities. It results in reduction of radius of an orbit of movement in Y at preservation of equality of values of centrifugal force and cosmological one. But then a motion pipe of a particle is not a cylindrical surface, its meridians have an inclination to the pipe axis, therefore the projection of cosmological force onto a meridian becomes distinct from zero. This projection both represents the force of gravitation and is equal to $F_\xi = -F\sin\chi = -\left(\hbar c/a_\infty^2\right)da/d\xi$, whence in approximation (21)

$$F_\xi = -\left(\hbar c/a_\infty\right)d\sqrt{\gamma}/d\xi = -mc^2\,d\sqrt{\gamma}/d\xi\,. \tag{22}$$

In spherically symmetric field the asymptotic decomposition of γ into a power series in $1/r$, where r is the radial co-ordinate (distance from the centre of gravitation from the point of view of the distant observer), has a form

$$\gamma = 1 - \left(r_g/r\right) + b_2\left(r_g/r\right)^2 + b_3\left(r_g/r\right)^3 + \cdots, \tag{23}$$

where $r_g = 2GM/c^2$ represents gravitational radius, G gravitational constant, M mass of an attractive body.

In (23) coefficient at the first power, as well as in the metric theory of gravitation, is chosen equal -1, in order, far away from the center of gravitation, gravitational potential was Newtonian one [10,11]. Substituting (23) in (22) yields

$$F_\xi = -\frac{GMm_\infty}{r^2}\left[1 - \left(2b_2 - \frac{1}{2}\right)\frac{r_g}{r} - \frac{3}{2}\left(b_3 + \frac{b_2}{2} - \frac{1}{8}\right)\left(\frac{r_g}{r}\right)^2 + \cdots\right].$$

The gravitation has the same effect onto rays of light how appropriate anisotropic medium, and the speed of light is described by the formula for ray velocity [13]:

$$\frac{1}{c_k^2} = \left(\frac{\sin\beta}{c_\perp}\right)^2 + \left(\frac{\cos\beta}{c_{||}}\right)^2, \tag{24}$$

where в is an angle between a direction of propagation of light and gradient of a field. Denoting projections of an element $d\sigma$ of a trajectory in X onto the direction of gradient of a field and onto perpendicular to it direction through $d\sigma_{||}$ and $d\sigma_\perp$, respectively, and substituting (24) in (8), one obtains

$$ds^2 = \gamma(cdt)^2 - \gamma\left(\frac{c}{c_{||}}d\sigma_{||}\right)^2 - \gamma\left(\frac{c}{c_{\perp}}d\sigma_{\perp}\right)^2.$$

Whence in neglect by the quantum corrections under conditions (21) one finds

$$ds^2 = \gamma(cdt)^2 - (1/\gamma)d\sigma_{||}^2 - (1/\gamma)d\sigma_{\perp}^2. \quad (25)$$

The metric (25) is described only by one function of co-ordinates – function γ. The centrifugal force $p_{\xi}v_{\varsigma}/R\cos\theta$, where R is the radius of curvature of a trajectory in X, is counterbalanced by component of force of gravity $-F_{\xi}\sin\beta$. From here one obtains

$$\tan^2\theta = \left(R/\sqrt{\gamma}\right)\left(d\sqrt{\gamma}/d\xi\right)\sin\beta. \quad (26)$$

The Lagrange function $L = -\hbar\sqrt{1-(\dot{r}/c_{||})^2 - (r\dot{\varphi}/c_{\perp})^2}$ in polar co-ordinates r, φ does not depend explicitly on φ, so that $\partial L/\partial\dot{\varphi} = const.$, whence one obtains the law of conservation of angular momentum $(c/c_{\perp})^2 rv\sin\beta = const.$ Substituting (21) yields $v(r/\gamma)\sin\theta\sin\beta = const.$ By means of this formula with due account of (15) it is possible to eliminate $\sin\beta$ or $\sin\theta$ in (26).

For introduction of coordinates with reference to (25) we may use an Einstein's equation for components of the Riccian tensor. In vacuum, this is $R_{00} = 0$. In spherically symmetric field, this equation for $\gamma = \exp(\nu)$ is reduced to $\nu'' + \nu'(2/r) = 0$ [10]. Its solution, satisfying the asymptotics (23), has a form $\nu = -r_g/r$, $b_2 = 1/2$. By this the metrics (25) in spherically symmetric field coincides in the post-Newtonian approximation with Shwarzshild's metrics in isotropic coordinates [10,11] and with the metrics of the relativistic theory of gravitation [13], but differs from both these metrics in the next approximation. This solution is obtained as well from a hypothesis, that the superposition of partial fields ν_j (i.e. of gravitational potentials) takes place for any components M_j of the total mass M, including infinitesimal ones, so that $\nu = \sum_j \nu_j$. Really, for $M_j = M/n$ $(r_{gj} = r_g/n)$ one has $\nu_j = -r_{gj}/r$, $\nu = \lim_{n\to\infty}(n\nu_j) = -r_g/r$. Given superposition corresponds to the principle of simplicity also and at any spatial distribution of masses. Anyway, for such static distribution of masses the replacement of exponential function $\exp\left[-\sum_j (r_{gj}/r_j)\right]$ by the first three terms of its power series expansion gives the metrics coincident with post-Newtonian metrics given in [11].

It is essential, that as distinct from others of multidimensional theories of gravitation, in the given approach based on the principle of simplicity, a compactification of space of extra dimensions is not a necessary. It is replaced here by presence of the cosmological force, confining particles in Compton vicinity of three-dimensional subspace (X). Here are compactificated not extra dimensions, but trajectories of elementary particles in space of extra dimensions. Existence of this force is not postulated. It follows from the principle of simplicity (specifically, from the principle of similarity of the basic properties of substance and light, according to which $mc^2 = h\nu$; the value of force is equal to $p_y c/a_{\infty} = m^2c^3/\hbar$) and from fact of existence of three-dimensional bodies. If such force would not exist, the particles would not be kept in a vicinity of three-dimensional subspace. Then for an explanation of existence of three-dimensional bodies it should with necessity involve a compactification of space of extra dimensions, despite of the problem of an explanation of its occurrence. The problem of compactification in multidimensional theories of gravitation arises because of impossibility of an explanation of existence of three-dimensional bodies in multidimensional space, without the consideration of mechanism of confining of particles in a small vicinity of three-dimensional subspace. In the given approach this problem does not arise.

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