Six-Dimensional Treatment of CPT Symmetry

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In distinct of standard formulation of the CPT theorem, in which the properties of particles and antiparticles, respectively, under direct and reverse flow of time are collated, in the six-dimensional treatment of CPT symmetry the properties of the same elementary particle are collated under direct and reverse flow of time. In this treatment the charges of particles and antiparticles are the same but the signs of the corresponding electrical and magnetic fields are defined by the sense of revolution of the particle or antiparticle in the extra dimensions space (in a circle of Compton radius). Under change of the flow of time, the sense of this revolution is changed on reverse one, that leads to the change of signs of the fields on opposite ones. By this the corresponding trajectories in the whole space occurs to be as reflected from a mirror. The motion of a particle along the helical line (of Compton radius) with revolution to the left (right), viewing in the direction of travel, is changed onto the motion along the mirror-reflected helical line with revolution to the right (left). The corresponding formulation of the theorem is following: If the flow of time is reversed, the particle moves in the whole space backward along the same trajectory as under direct flow of time. By this automatically the signs of the fields change on opposite ones, and the trajectory, viewing in the direction of travel, in the whole space occurs to be as reflected from a mirror, so that this particle acquires all properties of the antiparticle. The sign of charge may be regarded as nothing but a mark corresponding to positive or negative sense of revolution in the space of extra dimensions. The six-dimensional treatment of the Coulomb force of interaction between two charges is given. The electric force is due to motion of charges in the extra-dimensional subspace and is equal to correspondent Lorentz force.

The equation of dispersion is the same for acoustic waveguide, electromagnetic one, and de Broglie waves: $v_{ph}v_g = c^2$, where v_{ph} is the phase velocity, v_g group velocity, c speed of waves in a free medium (speed of sound in the first case and of light in two other cases). The main characteristic of any waveguide is that it has finite transverse dimensions. The dispersion of waves is due to just these dimensions. It indicates that the space with which we deal in experiments and observations is three-dimensional only approximately, but has a small (Compton) extra-dimension thickness.

The proposed treatment is based on the principle of simplicity [1] giving preference to that among competing hypotheses which is based on smaller number of postulates, that is, more simple. It rises from Einstein's statement "the nature saves on principles" and idea of F. Klein [2-4] on movement of particles with the speed of light in a multi-dimensional space. These ideas entered in that principle.

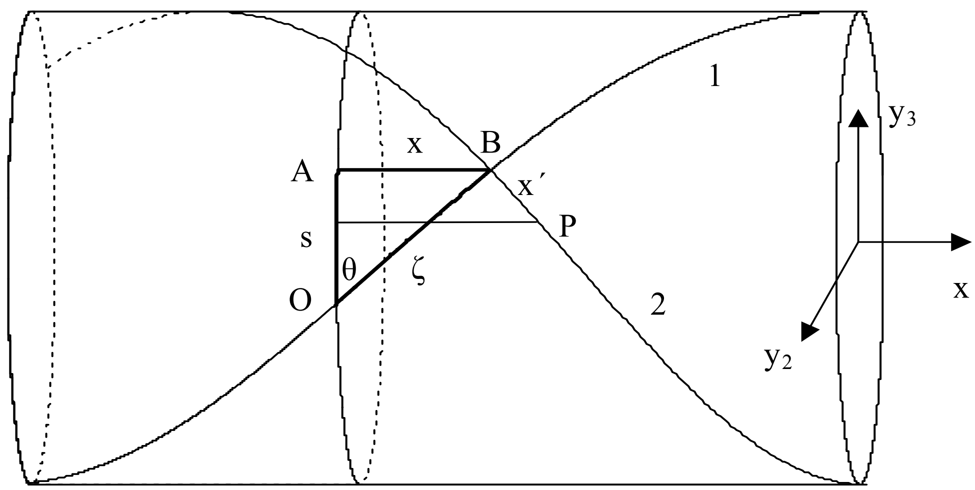
It is well known that the light and as well particles of substance have corpuscular as well wave properties of which examples are diffraction of electrons, when they represent as a wave, and photo electric emission, when photon represents as a particle. On this reason, following to the principle of simplicity, it is naturally to suppose that several basic properties of light and particles are similar. The basic property of light is its propagation with the same speed in any system of reference. Then as well elementary particles of substance must move with the same speed. It is impossible in three-dimensional space but possible in multi-dimensional one if positions of particles are recording by an observer in projection on three-dimensional space x_1, x_2, x_3 (X) which we shell consider as homogeneous and isotropic. By this, Newtonian insight extended on six-dimensional Euclidean space (R_6) with projection on three-dimensional space X give known relativistic results.

The whole space is supposed to be six-dimensional one, as only for it a simple interpretation of spin and isospin of electron and other elementary particles is possible. The first substantiation of six-dimensionality of space was given in [5], where universal physical constants are calculated.

Assume that for moving with speed of light in six-dimensional space R_6 elementary particles considered as material points, formulas of the Newtonian mechanics are applicable with appropriate chose of time (specifying below). The particles should be acting by a force (of cosmological nature), which is orthogonal to subspace X and keeping them in small vicinity of X. Without such force, withstanding centrifugal force, existence of macroscopic three-dimensional bodies in the Universe would be impossible. The positions of particles are fixing by an observer in the projection on subspace X. (More precisely, we use cosmological small site of X tangential to three-dimensional Universe as three-dimensional sphere in six-dimensional space with neglecting the curvature near this site).

The particle, which is at rest in a projection on X in an inertial frame of reference, moves with the speed of light c, in the simplest case, in a circumference in three-dimensional subspace Y adding up X until R_6, with the center of the circumference in X by $y_1 = y_2 = y_3 = 0$). In any other inertial frame of reference this particle is moving in a helical line located on a cylindrical surface (a motion tube) in R_6 with an axis in X.

By natural measure of the proper time of a particle is the number of its revolutions in the additional subspace Y about the axis of tube. Accordingly, we assume that the proper time of a particle is proportional to the number of such revolutions in Y or to the path length traveling in Y.



1 – the helical trajectory of a particle moving in six-dimensional space with the speed of light c along the cylinder surface of Compton radius $a = \hbar/(mc)$ with axis in the subspace X and directrix in the subspace Y

2 – the helical line of equal proper time of this particle. It passes through the particle perpendicularly to that helical trajectory. It moves along the same cylinder surface with velocity of de Broglie wave. Its pitch is equal to the de Broglie wavelength

Generally, the number of revolutions of a particle is proportional to $|\cos\theta|$, where θ is the angle of an inclination of a helical line, as shown in the figure. Therefore, if a particle makes one revolution per a proper time τ by clock of the observer "at rest", relative of which the particle moves along the tube with a speed $v = c\sin\theta$, where c is the speed of light, then it will take place per time $t = \tau/|\cos\theta|$. It is obvious that

$$\sin\theta = v/c,\ \cos\theta = \pm\sqrt{1-(v/c)^2}\ , \tag{1}$$

where upper sign is referred to the particle revolving about the axis of tube in positive sense and lower sign is referred to the antiparticle revolving in negative sense. Such a chose of sign corresponds to the following relation between lapses of proper time of a particle (or antiparticle) $d\tau$ and time of the observer at rest dt:

$$dt = \pm d\tau\Big/\cos\theta = d\tau\Big/\sqrt{1-(v/c)^2}\ . \tag{2}$$

In the frame of reference at rest (K), a particle moving with the speed of light c on the motion tube under angle θ to the direcrix of the tube has a component of the speed along the direcrix equal to $v\cos\theta$. According to (2) the proper time of a particle from the point of view of the observer at rest is proportional to $\cos\theta$ as well, so that the particle in the proper frame of reference (K') moves with the speed of light c as well.

A particle at rest in K, a particle moving with the speed of light c along the direcrix, displaces per proper time $d\tau$ in an interval ds equal to

$$ds = \pm c d\tau \ . \quad (3)$$

The momentum of this particle is a vector directed along the tangent to the direcrix at a point where this particle is placed at given time. The magnitude of this vector is mc being the product of mass m of the particle by its speed c. This is the momentum at rest in relativistic mechanics. The energy at rest E_0, according to definition, is the product of momentum and the speed of a particle: $E_0 = mc^2$. In the general case, the total momentum of a particle is the vector directed along the tangent to the helical trajectory. Its value p is the product of mass m of a particle by the relation of its path

$$d\varsigma = cdt \quad (4)$$

in R_6 to a proper time $d\tau$ expended for this path:

$$p = m\frac{d\varsigma}{d\tau} = \frac{mc}{|\cos\theta|} = mc\Big/\sqrt{1-(v/c)^2} \ . \quad (5)$$

This is relativistic formula for total momentum of a particle [6].

Projections p_x and p_y of a total momentum on the generatrix and directrix of a tube are equal to co-ordinate and temporal components of 4- momentum of a particle, respectively [6]:

$$p_x = \pm mc\tan\theta = mv\Big/\sqrt{1-(v/c)^2} \ , \quad p_y = \pm mc \ . \quad (6)$$

In the general case, $\theta \neq 0$ and the total energy of a particle E is the product of the total momentum p by the speed of movement c along a helical line:

$$E = pc = \frac{mc^2}{|\cos\theta|} = mc^2\Big/\sqrt{1-(v/c)^2} \ . \quad (7)$$

This value is the total relativistic energy of a particle. Note that the relation of the total energy to the total momentum of a particle occurs to be the same as for a photon. It is yet another common property of light and substance.

Let us assume that particles having charges of opposite signs revolve about the axis of motion tube in opposite senses. Particles and antiparticles have charges of opposite signs and revolve in opposite senses. For time undergoes a reversal, a particle would go back along its helical trajectory and hence revolve in opposite sense. This signifies that its charge has to change its sign, so that this particle has to transform to its antiparticle. In this case, the motion of such a particle will be as reflected in mirror. The sum of above properties is CPT symmetry.

The displacement of a particle in an interval ds along a directrix of a motion tube and respective turn through a central angle $d\phi = ds/a$ about the tube axis, where a is radius of the tube, is identical in any frame of reference, it is invariant. It is because an angle ϕ of a turn of a particle about the axis of the tube is independent on a velocity of an observer in X relative to this particle.

Denoting through dx, in the system of reference K, a projection of a displacement $d\varsigma$ of a particle on the surface of the tube on its generatrix and applying the Pithagorean theorem to the rectangular triangle OAB shown in figure, one obtains the expression for the interval: $(ds)^2 = (cdt)^2 - (dx)^2$. The projection of sides of that triangle on the trajectory of the particle gives

$$s\cos\theta + x\sin\theta = \varsigma \ . \quad (8)$$

Put initial conditions in the form $t = \tau = 0$ by $x = s = 0$. Then referring to (3) and (4) it follows:

$$s = \pm c\tau \ , \quad \varsigma = ct \ . \quad (9)$$

Substituting (1) and (9) into (8) gives the Lorentz transform for time:

$$\tau = \pm[t - (x/c)\sin\theta]/\cos\theta = \left[t - \left(xv/c^2\right)\right]\Big/\sqrt{1-(v/c)^2} \ .$$

Similar consideration applied to the system of reference K' with account for that the system K moves relative to considered particle with velocity $-v$ leads to the reversed transform: $t = \pm[\tau + (x'/c)\sin\theta]/\cos\theta = [\tau + (x'v/c^2)]/\sqrt{1-(v/c)^2}$, where x' is the co-ordinate along the line of equal proper time. To the transition from the system K to K' corresponds a turn throw an angle $-\theta$ about the origin $x = s = 0$ of co-ordinate net x, s on the surface of the motion tube, together with trajectories of particles on it. This turn transfer a helical trajectory in the directrix of the tube.

For a geometrical interpretation of rest Lorentz transforms, let us consider a trajectory of a particle moving along the tube with the same velocity v and intersecting at a time $t = 0$ the helical line $s\cos\theta + x\sin\theta = 0$ at arbitrary point P. In the system of reference K, trajectories inclined under the angle θ to the directrix are the lines of constant co-ordinate x' of the system K'. The co-ordinate $x' = BP$ is measured along the helical line describing by equation (8). The measuring is taken from the normal section of tube $x = vt = \varsigma\sin\theta$ until a section of which the particle P achieves at time t when x is the distance between P and the directrix OA. Projecting segments x', x, ς, and s on the generatrix and directrix, the trajectory of particle, and the helical line (along x') perpendicular to the trajectory one obtains by $\cos\theta > 0$:

$$x'\cos\theta + \varsigma\sin\theta = x,\ s\cos\theta + x\sin\theta = \varsigma,\ \varsigma\cos\theta - x'\sin\theta = s,\ x\cos\theta - s\sin\theta = x'.$$

Dividing these equalities throw by $\cos\theta$ and eliminating s, ς and θ by means of (1) and (9) according to which, in considered case, $s = c\tau$, $\varsigma = ct$, $\sin\theta = v/c$, $\cos\theta = \sqrt{1-(v/c)^2}$, one may easy obtain the Lorentz transforms in the standard form.

The proper length of moving rigid scale is the difference of co-ordinates x' of its ends. In the system K, it is equal to the length of a segment of the helical line perpendicular to the trajectories of particles moving with this segment between normal sections of the motion tube corresponding to those ends. It is a segment of the line of equal time in the system K'. The length of the same scale in the system at rest K is the difference of co-ordinates x of its ends. It is equal to the distance along the generatrix between those normal sections that is in $1/\cos\theta$ less than the proper length.

Thus, the Lorentz contraction of moving scales is a result of projection of lengths in multi-dimensional space on three-dimensional space. Non-simultaneity of spatially spaced events in one system of reference with simultaneity in another is explained by non-parallelism of helical lines of equal time in systems of reference moving one relative another.

Above interpretation of the formula (2) holds as well for curve axis of a motion tube because in any case all normal sections of such tube are perpendicular to any directions in the subspace X to which belongs the axis of a tube.

The energy of a photon is equal to $h\nu$, where ν is frequency of light, h the Plank constant. By virtue of a principle of similarity of the basic properties of substance and light concretizing the principle of simplicity, the rest energy of a particle may be represented as a quantum of energy $h\nu$, so that

$$mc^2 = h\nu. \tag{10}$$

Unique and natural frequency ν for a particle of substance is the frequency of its revolutions in extra-dimensional subspace Y. On the other hand, the particle moves with the speed of light along the directrix of the motion tube, whence $2\pi a = c/\nu$, where a is radius of the tube. Eliminating ν from this equality and (10), one finds $2\pi a = h/mc$, $a = \hbar/mc$, that is the length of directrix is equal to Compton wavelength.

Another helical line placed on the same tube perpendicularly to helical trajectory of a particle and passes through the particle, is the line of equal proper time of the system K'. This helical line moves along the tube with velocity of de Broglie wave $V_\phi = c/\sin\theta = c^2/v$, where v is velocity of the particle in the subspace X. The pitch ℓ of this helical line is equal to the de Broglie wavelength $\ell = \frac{2\pi a}{|\tan\theta|} = \frac{h}{mc|\tan\theta|} = \frac{h}{p_x} = \frac{h}{|mv|}\sqrt{1-(v/c)^2}$, as it is seen from (6) and above figure. The angle co-ordinate s/a of the helical line describing by (8) and (9) is equal to $\frac{s}{a} = \frac{\varsigma}{a\cos\theta} - \frac{x}{a}\tan\theta = $ $= \left(t\frac{c}{\cos\theta} - x\tan\theta \right)\frac{mc}{\hbar}$. Whence and from (6) and (7) is seen that s/a is equal to the phase of de Broglie wave $\pm\left[Et - p_x(x/\hbar)\right]$. In the place of position of the particle $x = vt$ this phase is an angle of turn of itself particle on the motion tube. The function $\exp(i\,s/a)$ satisfies the Klein-Gordon equation.

The proper moment of momentum $\boldsymbol{S}$ of a particle is a vector product of the proper momentum and radius vector of this particle. The component of the radius vector and the component of velocity of the particle on the axis of the motion tube are perpendicular to the plane of revolving in Y and therefore do not give any contribution in $\boldsymbol{S}$. Hence for a particle moving in six-dimensional space along a helical line but consequently in a straight line in a projection on X, $\boldsymbol{S}$ is a vector product of projection of momentum and radius vector of this particle on Y. In this case, the magnitude of momentum $\boldsymbol{S}$ becomes $S = |\boldsymbol{S}| = |p_y a| = mc\hbar/mc = \hbar$. This formula remains some arbitrariness in the orientation of vector $\boldsymbol{S}$ in six-dimensional space: it may be oriented in any direction in four-dimensional subspace perpendicular to the plane of revolving in Y. In the general case, vector $\boldsymbol{S}$ has four nonzero components along directions perpendicular each to other and the plane of revolving of the particle in Y. In the case of revolving in the plane y_2, y_3, such components are S_1, S_2, S_3, S_4 along the axes x_1, x_2, x_3, y_1, respectively, and $S = \left(S_1^2 + S_2^2 + S_3^2 + S_4^2\right)^{1/2} = \hbar$. Components S_1, S_2, S_3 are components of spin of the particle, S_4 is a projection of isospin of the particle. Thus, spin and isospin are the projections on X and Y, respectively. By (6), p_y is independent on velocity v. Hence spin and isospin are independent on velocity v also and do not subjected to the Lorentz transforms.

Vector $\boldsymbol{S}$ remaining perpendicular to the plane of revolving of the particle has three degree of freedom and may be oriented in arbitrary manner relative to those co-ordinate axes. To particles with spin one half corresponds uniform distribution of components of the vector over above four axes perpendicular each to other and the plane of revolving in Y. Then these components are equal to $+\hbar/2$ or $-\hbar/2$, and the sum of squares of these components in X is equal to $(3/4)\hbar^2$. In quantum mechanics it is "total" (in three-dimensional space) square of the proper momentum of a particle.

To last case orientations of vector $\boldsymbol{S}$ obtained from previous orientations through allowable turns retaining one or two given components invariable are referred as well. So, if one of components of the vector in X and one component in Y have a fixed value $+\hbar/2$ or $-\hbar/2$, then the vector retain a possibility to turn about two correspondent axes. In this case, two non-fixed components will not have of specific values (it is ordinary situations in quantum mechanics, where absence of fixation of quantities is rather the exclusiveness than a rule). For equal allowed probabilities of orientations of that vector, means-square components mentioned above are equal to $\hbar/2$. Change of a direction of revolving of a particle about the axis of the motion tube on the opposite sense as well changes the signs of the components on opposite and corresponds to the transition to antiparticle.

The relations of Heisenberg uncertainty are due to uncertainty of co-ordinates and moments of a particle in Y. In fact, let the directix of a motion tube of a particle is displaced in the plane y_2, y_3. Then projections of the momentum of a particle on axes y_2 and y_3 and coordinates of the particle along this axes are equal to

$p_{y2} = -mc\sin\varphi$, $p_{y3} = mc\cos\varphi$, $y_2 = \frac{\hbar}{mc}\cos\varphi$, $y_3 = \frac{\hbar}{mc}\sin\varphi$, where ϕ is the angle of a turn of the particle about the axis of tube reckoned from the axis y_2. Average over ϕ values of coordinates and projections of the momentum are equal to zero but their mean-square values are equal to

$\left\langle y_2^2 \right\rangle = \left\langle y_3^2 \right\rangle = \frac{1}{2}\left(\frac{\hbar}{mc}\right)^2$, $\left\langle p_{y2}^2 \right\rangle = \left\langle p_{y3}^2 \right\rangle = \frac{1}{2}(mc)^2$, whence one finds seeking relations $\left\langle p_{y2}^2 \right\rangle \cdot \left\langle y_2^2 \right\rangle = \left\langle p_{y3}^2 \right\rangle \cdot \left\langle y_3^2 \right\rangle = \hbar^2/4$.

It is of interest, why the values of the proper momentum and its components in X and Y, that is spin and isospin, are independent on mass of an elementary particle? In six-dimensional treatment the answer is obvious: the momentum is proportional to this mass but the radius of the Compton orbit in Y for this particle is inversely proportional to this mass, and therefore the product of momentum and radius of the Compton orbit is independent on this mass.

The proper magnetic moment $\boldsymbol{\mu}$ of a charged elementary particle is defined similarly to the proper moment of momentum $\boldsymbol{S}$ accordingly to the known formula of electrodynamics [7]: $\boldsymbol{\mu} = \frac{e}{2c}[\boldsymbol{Rc}]$, where $\boldsymbol{R}$ is six-dimensional radius vector of the particle, $\boldsymbol{c}$ vector of its velocity in Y. Since a contribution in this vector product gives only the projection $\boldsymbol{a}$ of the radius vector $\boldsymbol{R}$ on subspace Y, one finds $\boldsymbol{\mu} = \frac{e}{2c}[\boldsymbol{ac}]$. Whence, accounting for mutual perpendicularity of vectors $\boldsymbol{a}$ and $\boldsymbol{c}$ as well equalities $|\boldsymbol{a}| = a$ and $|\boldsymbol{c}| = c$, one finds the magnitude μ of the proper moment $\boldsymbol{\mu}$ of the particle which occurs to be equal to the Bohr magneton:

$$\mu = \frac{|e|a}{2} = \frac{|e|\hbar}{2mc} = \mu_B \, . \tag{11}$$

In the simplest case, when the vector $\boldsymbol{\mu}$ has not components in subspace Y, the components of $\boldsymbol{\mu}$ in X defines a three-dimensional vector, of which magnitude is equal to the Bohr magneton.

A projection of the magnetic moment onto arbitrary chosen direction (called the axis of quantization) in subspace X may have a fixed value only in the case when the projection of the proper moment of momentum has a fixed value as well. In this case, according to (11) $\mu_x = \pm\mu_B$. At uniform distribution of components of the proper moment of momentum over four axes which perpendicular each to other and a plane of revolving of a particle in Y, in considered case are $S_x = \pm\, mca/2$, that equals $+1/2$ or $-1/2$ (in units of $\hbar$). Whence $\mu_x / S_x = e/mc$ in accordance with the experiment of Stern and Gerlach.

The six-dimensional treatment of considered above and other physical values and phenomena stated in [8-12].

In the general case, the moment of momentum has four nonzero components along directions perpendicular each to other and a plate of revolving of a particle. Therefore the theory of spin and isospin must use explicitly or implicitly four co-ordinates and four projections of vectors on the axes of that co-ordinates. The total moment of momentum $\boldsymbol{M}$ in R_6 is the vector product of the total momentum $\boldsymbol{p_x} + m\boldsymbol{c}$ and radius-vector $\boldsymbol{r} + \boldsymbol{a}$ of a particle in R_6, where $\boldsymbol{p_x}$ and $\boldsymbol{r}$ denote momentum and radius vector in X, $m\boldsymbol{c}$ and $\boldsymbol{a}$ momentum and radius vector in Y. Moment $\boldsymbol{M}$ is four-dimensional vector perpendicular to the plane of revolving of a particle (in Y). On average over a period of revolution about the axis of the tube, the cross terms disappear and then $\boldsymbol{M} = \boldsymbol{L} + \boldsymbol{S}$, where $\boldsymbol{L}$ is the orbital moment in X, and $\boldsymbol{S} = [\,\boldsymbol{a}\; m\boldsymbol{c}\,]$ the spin-isospin moment of revolving in Y. Three components of $\boldsymbol{S}$ represent the spin projections S_1, S_2, S_3 on X, and the component on Y represents isospin S_4. Hence, on account of the mutual perpendicularity of vectors $\boldsymbol{a}$ and $\boldsymbol{c}$, and equalities $|\boldsymbol{a}| = a$, $|\boldsymbol{c}| = c$, one obtains $S = \hbar$, $S_1^2 + S_2^2 + S_3^2 + S_4^2 = \hbar^2$.

At uniform distribution of components on four axes of co-ordinates, which are perpendicular to the plane of revolving in Y, one finds $|S_j| = \hbar/2$, $j = 1,2,3,4$; $S_1^2 + S_2^2 + S_3^2 = 3\hbar^2/4$.

The disposition of two electrons on the opposite sides of the same tube of motion has energetic advantage. By this the distance between them in the whole space is equal to $R=\sqrt{r^2+4a^2}$ where r is the distance between projections of the particles onto X, a is the distance from the axis of their revolution in Y. The tube radius is depended on r and tending asymptotically to $a_\infty=\hbar/(mc)$ with increasing of r, m and c being mass of particle and speed of light at infinity, respectively. By such a revolution with the shift in phase π between two particles, the Coulomb force of their repulsion in the whole space is equal to e^2/R^2 where e is the charge of electron. Projections of this force onto subspaces X and Y are $F_\parallel=(e^2/R^2)\sin\chi$ and $F_\perp=(e^2/R^2)\cos\chi$, respectively, where $\sin\chi=r/R$, $\cos\chi=2a/R$, so that $F_\parallel=e^2r/R^3$, $F_\perp=2e^2a/R^3$. The force $F_\perp$ reacts against centripetal force $F_0=mc^2/a_\infty$. On this cause the radius of revolution a is a little in excess of the tube radius a_∞ at infinity.

The energy at rest and centrifugal force in Y, as in the six-dimensional theory of gravitation [10, 11], are equal to $E_0=p_y c_\varsigma=mc^2\sqrt{\gamma}$ and $F_c=p_y c_\varsigma/a=E_0/a$, respectively, where c_ς is speed of the particle on the motion tube, $p_y=\hbar/a$ momentum at rest, $\sqrt{\gamma}=c_\varsigma\, a_\infty/ca$, so that $c_\varsigma=ca\sqrt{\gamma}/a_\infty$.

The balance of forces in Y is $F_0=F_\perp+F_c$. Referring to the relation $e^2/mc^2=\alpha a_\infty$ (this is the classical radius of electron, α is the fine structure constant) and introducing $z=a/a_\infty$, this balance of forces may be represented as

$$\sqrt{\gamma}=z-2\frac{\alpha}{\rho^3}z^2, \tag{12}$$

where $\rho=\sqrt{(r/a_\infty)^2+4z^2}$, $r=a_\infty\sqrt{\rho^2-4z^2}$, r is the three-dimensional distance.
Under the condition $c_\varsigma a=ca_\infty$ of conservation of angular moment, one finds

$$c_\varsigma=c/z, \qquad \sqrt{\gamma}=1/z^2. \tag{13}$$

If $r=0$, then $\rho=2z$ and by (12) and (13) one obtains the equation $z^3-\frac{\alpha}{4}z-1=0$, whence

$$z=\sqrt[3]{\frac{1}{2}+\frac{1}{2}\sqrt{1-\frac{1}{2}\left(\frac{\alpha}{6}\right)^3}}+\sqrt[3]{\frac{1}{2}-\frac{1}{2}\sqrt{1-\frac{1}{2}\left(\frac{\alpha}{6}\right)^3}}=1+\frac{\alpha}{12}-\frac{1}{3}\left(\frac{\alpha}{12}\right)^3+\frac{1}{3}\left(\frac{\alpha}{12}\right)^4+\cdots \tag{14}$$

If electrons are ejected on head one to other with the same speed v in X, the principle of energy yields

$$\frac{2}{\beta_\infty}mc^2=\frac{2}{\beta}mc_\varsigma^2+2\frac{mc^2}{a_\infty}(a-a_\infty)+\frac{e^2}{R}\left(1-\frac{v^2}{c_\varsigma^2}\right)^{-1}, \tag{15}$$

where $\beta=\sqrt{1-(v/c_\varsigma)^2}$, $\beta_\infty=\sqrt{1-(v_\infty/c)^2}$, v_∞ is the speed at infinity. The left side of (15) is the total energy of the pair particles at infinity. The first term in the right side of this equation is the total energy of the pair under consideration at given r. The second term is equal to the work against the centripetal force $F_0=mc^2/a_\infty$ at approach of the particles. The third term is an electric potential created by the charge coming nearer, in the position point of other charge.

By (12), (13), the equation (15) may be represented in the form

$$\frac{1}{\beta_\infty}=\frac{1}{\beta z^2}+\frac{\alpha}{2\rho}\left(1-\frac{v^2}{c^2}z^2\right)^{-1}+z-1\,. \tag{16}$$

Whence by $v=0$ from (16) one has $\frac{1}{\beta_\infty}=\frac{1}{z^2}+\frac{\alpha}{2\rho}+z-1$. If as well $r=0$, then

$$\frac{1}{\beta_\infty}=\frac{1}{z^2}+\frac{\alpha}{4z}+z-1=2z-1\,, \tag{17}$$

and according to (14) the kinetic energy at infinity is equal to

$mc^2\left(\frac{1}{\beta_\infty}-1\right)=mc^2 2(z-1)=mc^2 1.216225\times10^{-3}$, that for electron is equal to 621.485 eV.

The same result is obtained by equating the difference of total energies of the pair of particles, calculated at $r=\infty$ and $r=0$, to the work against the centripetal force F_0 and electric repulsive force e^2/R^2 at change r from infinity up to zero:

$$mc^2\frac{2}{\beta_\infty}-2mc_\varsigma^2(r=0)=2\frac{mc^2}{a_\infty}(a-a_\infty)+\int_{2a}^{\infty}\frac{e^2}{R^2}dR \tag{18}$$

at c_ς and z given by the formulas (13) and (14). Referring to $e^2=mc^2\alpha a_\infty$, after elementary integration (18) is reduced to (17).

Applying the Biot – Savart formula to six-dimensional space, the total magnetic field of the charge at rest in X is defined at the distance R from the charge e as $\boldsymbol{H}_{\text{tot}}=\frac{e}{cR^2}[\boldsymbol{c}\,\boldsymbol{R}_0]$ where $\boldsymbol{R}_0$ is the unit vector directed from the charge to the point of observation, $\boldsymbol{c}$ the velocity of the charge. For R being the distance between two electrons $\boldsymbol{R}_0=\boldsymbol{r}_0\sin\chi+\boldsymbol{a}_0\cos\chi=\boldsymbol{r}_0(r/R)+\boldsymbol{a}_0(2a/R)$,

$$\boldsymbol{H}_{\text{tot}}=\frac{e}{R^2}[\boldsymbol{c}_0\boldsymbol{R}_0]=\frac{e}{R^2}\{[\boldsymbol{c}_0\,\boldsymbol{r}_0]\frac{r}{R}+[\boldsymbol{c}_0\,\boldsymbol{a}_0]\frac{2a}{R}\}, \tag{19}$$

where $\boldsymbol{r}_0$ is unit vector along radius vector $\boldsymbol{r}$ in X, $\boldsymbol{a}_0$ unit vector along radius vector of the charge e in the plane of revolution in Y, and $\boldsymbol{c}_0$ is unit vector along velocity $\boldsymbol{c}$.

Let us show that the Coulomb force of interaction between the two charges (e and e') is the Lorentz force acting on this charges as moving in Y. Referring to (19) this force is equal to

$\boldsymbol{f}=\frac{e'}{c}[\boldsymbol{c}'\boldsymbol{H}_{\text{tot}}]=\frac{e'e}{cR^2}\{[\boldsymbol{c}'[\boldsymbol{c}_0\,\boldsymbol{r}_0]]\frac{r}{R}+[\boldsymbol{c}'[\boldsymbol{c}_0\,\boldsymbol{a}_0]]\frac{2a}{R}\}.$

Whence, with account of that for two interacting electrons $\boldsymbol{c}'=-\boldsymbol{c}$,

$\boldsymbol{f}=-\frac{e'e}{R^2}\{[\boldsymbol{c}_0[\boldsymbol{c}_0\boldsymbol{r}_0]]\frac{r}{R}+[\boldsymbol{c}_0[\boldsymbol{c}_0\boldsymbol{a}_0]]\frac{2a}{R}\}$. Revealing the triple vector products and taking into account mutual perpendicularity of involved vectors and that in the case under consideration $e'=e$, one obtains $\boldsymbol{c}_0[\boldsymbol{c}_0\,\boldsymbol{r}_0]]=-\boldsymbol{r}_0$, $[\boldsymbol{c}_0[\boldsymbol{c}_0\boldsymbol{a}_0]]=-\boldsymbol{a}_0$, $\boldsymbol{f}=\frac{e^2}{R^3}r\,\boldsymbol{r}_0+\frac{e^2}{R^3}2a\,\boldsymbol{a}_0$.

In the last formula the first term represents the projection of the Coulomb force onto X, the second term is its projection onto Y. Their magnitudes are equal to $F_{||}$ and $F_\perp$, respectively. From this is seen that electric forces are due to the moving of charges in subspace Y, in distinct of that usual magnetic forces are caused by moving of charges in the same subspace X. The force $F_{||}$ is equal to zero at $r=0$. This is the point of indifferent equilibrium, near which electrons may be slow moving comparatively long time if they were ejected on head one to other with original energy 621.485 eV .

Under change of the flow of time, the sense of revolution of particles in Y is changed on reverse one, that leads to the change of signs of the fields on opposite ones. By this the corresponding trajectories in the whole space occurs to be as reflected from a mirror. The motion of a particle along the helical line (of Compton radius in Y) with revolution to the left (right), viewing in the direction of travel, is changed onto the motion along the mirror-reflected helical line with revolution to the right (left). The sign of charge may be regarded as nothing but a mark corresponding to that or other (positive or negative) sense of revolution of a particle in the space of extra dimensions. In distinct of standard formulation of the CPT theorem, in which the properties of particles and antiparticles, respectively, under direct and reverse flow of time are collated, in the six-dimensional treatment of CPT symmetry the properties of the same elementary particle are collated under direct and reverse flow of time. In this treatment the charges of particles and antiparticles are the same but the signs of the corresponding electrical and magnetic fields are defined by the sense of revolution in the extra dimensions space. The corresponding formulation of the theorem is following: If the flow of time is reversed, the particle moves in the whole space backward along the same trajectory as under direct flow of time. By this automatically the signs of the fields change on opposite ones, and the trajectory, viewing in the direction of travel, in the whole space occurs to be as reflected from a mirror, so that this particle acquires all properties of the antiparticle.

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